ECE 313 (Section B)
In-Class Project 3 - Thursday, October 10

Write the names and NetIDs of your group members here:

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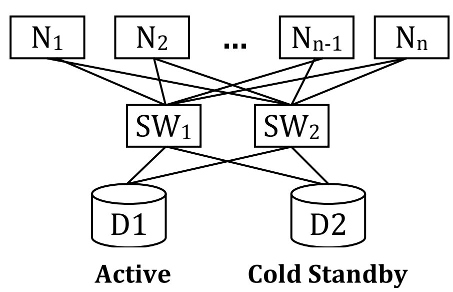
The following diagram shows the schematic of transaction clearing of a financial exchange system. Assume that the clearing system consists of three computing nodes (n = 3), dual redundant switches, and two disk arrays. In order for the clearing system to function properly *at least 2 out of 3* computing nodes are required to function (a TMR system) and *at least one* of the redundant switches should work. The disk arrays function as an *active-standby pair*: if the first disk fails, the second disk will be activated, i.e. the second disk is standby idle until the first disk fails. Assume that the links and the voter for TMR are perfect, the disk failures are always detected, the recovery to second disk is instantaneous, and if the fist disk fails at time τ, the second one fails at time $t - \tau$.



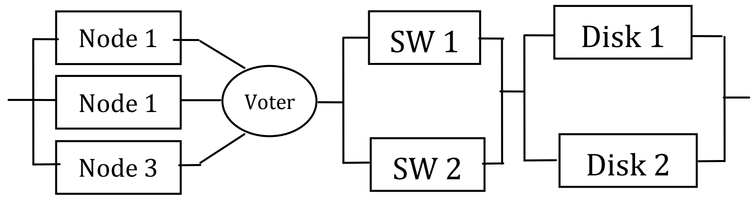
The lifetimes (time to failures) of each of the components in the system are exponentially distributed with the following parameters:

- Computing Nodes: λ_c
- Switches: λ_s
- Disk arrays: λ_a

a) Draw the reliability diagram of the whole system.

b) For each of the following subsystems, write the CDF for the time to failure (T), and from it derive the reliability function. Use the relation $E[T] = \int_0^\infty R(t)dt$ to calculate the mean time to failure (MTTF):
   - Three computing nodes
   - Two switches
   - Two disk arrays

Solution: The computing nodes, switches, and disk array subsystems are in series with each other. So the reliability block diagram of the system is drawn as follows:



So for each of the components we have:

$$f(t) = \lambda e^{-\lambda t} \quad , \quad F(t) = 1 - e^{-\lambda t} \;\Rightarrow\; R(t) = e^{-\lambda t}$$

And the MTTF = $1/\lambda$.

For each of the subsystems, we have:

– ***n* computing nodes:** ⇒ $R_{TMR}(t) = 3R_c^2(t) - 2R_c^3(t) = 3(e^{-\lambda_c t})^2 - 2(e^{-\lambda_c t})^3$

$$MTTF = E[T] = \int_0^\infty R_{TMR}(t)dt = \int_0^\infty \left(3e^{-2\lambda_c t} - 2e^{-3\lambda_c t}\right)dt$$

$$= \frac{3}{2\lambda_c} - \frac{2}{3\lambda_c} = \frac{5}{6\lambda_c}$$

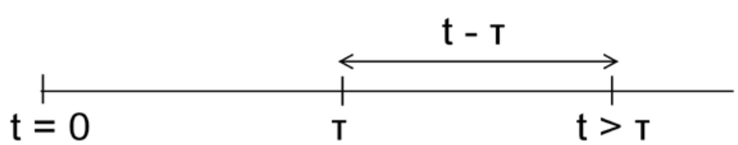
– **Two switches:** ⇒ $R_{2SW}(t) = 1 - (1 - R_S(t))^2 = 2R_S(t) - R_S^2(t) = 2(e^{-\lambda_s t}) - (e^{-\lambda_s t})^2$

$$MTTF = E[T] = \int_0^\infty 2e^{-\lambda_s t} - e^{-2\lambda_s t} dt$$

$$= \frac{2}{\lambda_s} - \frac{1}{2\lambda_s} = \frac{3}{2\lambda_s}$$

– **Two disk arrays:**

If we assume that the first disk array fails at some time τ, then the lifetime of second disk array starts at time τ, and it fails at a time $t > \tau$. So the lifetime of the second disk will be $t - \tau$:



The probability density function for the failure of the first disk array is:

$$f_1(t) = \lambda_a e^{-\lambda_a t}, 0 < \tau < t$$

Given that the first disk array must fail for the lifetime of the second disk array to start, the density function of the lifetime of the second disk array is conditional, given by:

$$f_2(t \mid \tau) = \begin{cases} \lambda_a e^{-\lambda a(t-\tau)}, & 0 < \tau < t \\ 0 & , \tau > t \end{cases}$$

Then we define the system failure as a function of t and τ, using the definition of conditional probability: $\varphi(t,\tau) = f_1(\tau) f_2(t \mid \tau)$

The marginal density function of *f(t)* (failure of system composed of 2-disk arrays) is:

$$f(t) = \int_0^t \varphi(t,\tau)\, d\tau = \int_0^t {\lambda_a}^2\, e^{-\lambda_a t} d\tau = {\lambda_a}^2 e^{-\lambda_a t}. [1]_0^t = {\lambda_a}^2 t e^{-\lambda_a t}$$

So the reliability function *R(t)* for the two disk array system will be calculated as follows:

$$R_{2disks}(t) = 1 - \int_0^t f(t) dt = \int_0^t {\lambda_a}^2 t e^{-\lambda_a t} dt$$

Integrating by parts, we get:

$$R_{2disks}(t) = (1 + \lambda_a t) e^{-\lambda_a t}$$

$$MTTF = E[T] = \int_0^\infty (1 + \lambda_a t) e^{-\lambda_a t} dt = \int_0^\infty e^{-\lambda_a t} dt + \int_0^\infty \lambda_a t e^{-\lambda_a t} dt = \frac{1}{\lambda_a} + \frac{1}{\lambda_a} = \frac{2}{\lambda_a}$$