ECE 313 (Section B)
In-Class Project 2 - Tuesday, September 24

Write the names and NetIDs of your group members here:

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Part 1 - Consider the experiment of tossing two dice. The sample space S is defined as: $S = \{(i,j) \mid 1 \le i,j \le 6\}$. Assume all the sample points have the equal probability of 1/36.

a) Let:
A = "The first die results in a 1, 2, or 6."
B = "The first die results in a 3, 4, or 5."
C = "The sum of the two faces is 9."

Write the events $A, B, \text{ and } C$ and calculate their probability.

Also write $A \cap B \;\; A \cap C \text{ and } A \cap B \cap C$ and calculate their probabilities

Determine if: $P(A \cap B) = P(A)P(B)$
$P(A \cap C) = P(A)P(C)$

Determine $P(A \cap B \cap C) = P(A)P(B)P(C)$

What you can say about exclusiveness and independence of these events?

b) Now repeat part (a) for the following events:
A = "The first die results in a 1 or 2."
B = "The second die results in a 4 or 5."
C = "The sum of the two faces is 6."

Part 2 - A telephone call may pass through a series of trunks before reaching its destination. If the destination is within the caller's own local exchange, then no trunks will be used. Assume p is the probability of reaching to destination. Let X the number of trunks used to reach to a destination, which is a modified geometric random variable with parameter p. Define Z to be the number of trunks used for a call directed to a destination outside the caller's local exchange. Given that a call requires at least three trunks, what is the conditional pmf of the number of trunks required?

a) Find the set of values that X might take and write its pmf.

b) Find the set of values that Z might take and write its pmf.

c) Write the expression for conditional probability and simplify it as much as you can

Solution:

Part 1 -

a) For events A, B, and C, we have:

A = {(1,1), (1,2), ..., (1,6), (2,1), (2,2), ..., (2,6), (6,1), (6,2), ..., (6,6)} => P(A) = 1/2
B = {(3,1), (3,2), ..., (3,6), (4,1), (4,2), ..., (4,6), (5,1), (5,2), ..., (5,6)} => P(B) = 1/2
C = {(3,6), (4,5), (5,4), (6,3)} => P(C) = 4/36 = 1/9

For the intersections we have:

$$A \cap B = \phi \Rightarrow P(A \cap B) = 0$$
$$A \cap C = \{(6,3)\} \Rightarrow P(A \cap C) = 1/36$$
$$A \cap B \cap C = \phi \Rightarrow P(A \cap B \cap C) = 0$$

So we have:
$$P(A \cap B) = 0 \neq P(A)P(B) = 1/4$$
$$P(A \cap C) = 1/36 \neq P(A)P(C) = 1/18$$
$$P(A \cap B \cap C) = 0 \neq P(A)P(B)P(C) = 1/36$$

$A \cap B = \phi$ **=> A & B are mutually exclusive**
$P(A \cap B) \neq P(A)P(B)$ **=> A & B are not pair-wise independent**

$A \cap C \neq \phi$ **=> A & C are not mutually exclusive**
$P(A \cap C) \neq P(A)P(C)$ **=> A & C are not pair-wise independent**

$B \cap C \neq \phi$
$P(A \cap B \cap C) \neq P(A)P(B)P(C)$ **=> A & B & C are not mutually independent**

b) A = {(1,1), (1,2), ..., (1,6), (2,1), (2,2), ..., (2,6)} => P(A) = 1/3
B = {(1,4), (2,4), ..., (6,4), (1,5), (2,5), ..., (6,5)} => P(B) = 1/3
C = {(1,5), (2,4), (3,3), (4,2), (5,1)} => P(C) = 5/36
For the intersections we have:

$$A \cap B = \{(1,4),(1,5),(2,4),(2,5)\} \Rightarrow P(A \cap B) = 4/36 = 1/9$$
$$A \cap C = \{(1,5),(2,4)\} = B \cap C \Rightarrow P(A \cap C) = P(B \cap C) = 2/36 = 1/18$$
$$A \cap B \cap C = A \cap C = B \cap C \Rightarrow P(A \cap B \cap C) = 1/18$$

$P(A \cap B) = 1/9 = P(A)P(B)$ **=> A & B are pair-wise independent**
$P(A \cap C) = 1/18 \neq P(A)P(C) = 5/108$ **=> A & C are not pair-wise independent**
$P(A \cap B \cap C) = 1/18 \neq P(A)P(B)P(C) = 1/36$ **=> ABC not mutually independent**

Part 2 -

a) X can take any values of {0, 1, 2, 3, …}, so the pmf of X is:

$$P_X(i) = P(X = i) = p(1-p)^i,\ i = 0,1,2,\ldots.$$

b) Z is the number of trunks used for a call directed to a destination outside the caller's local exchange, so at least one trunk will be used and Z takes values of {1, 2, 3, …}, Since X has modified geometric distribution, then Z has geometric distribution with pmf: $P_Z(k) = P(Z = k) = p(1-p)^{k-1}, k = 1,2,\ldots$

c) The conditional probability is required for

$$P(Z = k | Z \geq 3) = \frac{P(Z = k \; and \; Z \geq 3)}{P(Z \geq 3)}$$

$$= \begin{cases} \dfrac{p(1-p)^{k-1},}{(1-p)^{3-1}} & k \geq 3 \\ 0, & otherwise \end{cases}$$

$$or = (p(1-p)^{k-3}, k \geq 3.$$

Note that we are not shifting the origin of measurement and therefore the use of the memory-less property to obtain the answer is incorrect.