Mini Project 3

Binary Hypothesis Testing for Real-time Patient Monitoring

ECE 313 – Section B
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Mini Project 3: Description

- Remember Mini Project 2 where you analyzed three correlated physiological signals collected from the biomedical sensors attached to the patient’s body:
  - S1: Heart Rate (HR)
  - S2: Pulse Rate (PR)
  - S3: Respiration Rate (RESP)
- The signals were passed through three processing units (P1, P2, and P3, respectively), called threshold functions, to detect patient abnormalities.
- Each threshold function will generate an alarm whenever a data sample of the corresponding signal exceeds a pre-defined threshold. A “1” on the output of each function indicates an alarm and a “0” corresponds to absence of an alarm.

- Suppose that you are going to design two threshold functions the following pairs of signals and use MAP and ML decision rules to generate alarms.
  - (Heart Rate, Pulse Rate)
  - (Heart Rate, Respiration Rate)
- Assume that the three signals are independent.
- Unlike the previous problem there is no fusion unit here.

Mini Project 3: Task 0

- **Task 0:**

- Load the provided data set into MATLAB: *load patient_data.mat*

Similar to Mini Project 2:

- *patient_data* consists of 2 variables: *data* and *golden_alarms*:
- *data* is an array of 3 rows and 30,000 columns
  - Each of the rows 1 to 3 respectively correspond to one of the signals:
    - Heart Rate (*HR*), Pulse Rate (*PR*), Respiration Rate (*RESP*)
  - Each column corresponds to a data sample

- *golden_alarms* is a binary vector of 3,000 columns
  - Each column corresponding to a 10-sample interval.

Mini Project 3: Task 1

a) <u>Coalescing of data by averaging</u>: Coalesce the data samples for each of the variables *HR*, *PR*, and *RESP* by calculating the average of data samples over fixed intervals of 10 samples. i.e., for *HR* we calculate the average of all the 10 data samples in each interval and save it as the representative *HR* value for that interval, similarly for *PR*, and *RESP* variables.

   Hint 1: Save the results as three new vectors of 3,000 columns (*HR_AVG*, *PR_AVG*, and *RESP_AVG*).

   Hint 2: The average of the samples in an interval might not be an integer, use the *floor* function in MATLAB to round it to an integer value.

b) Divide each of the *HR_AVG*, *PR_AVG*, *RESP_AVG*, and *golden_alarms* vectors to two equal sections of <u>*training*</u> and <u>*testing*</u> and save them in 8 new vectors of 1500 columns each.

   Hint 1: For example, for *HR_AVG* and *golden_alarms*, we have the new variables:

   HR_AVG_training = HR_AVG(1:1500) -- *HR_AVG_testing = HR_AVG(1501:3000)*

   golden_training = golden_alarms(1:1500) -- *golden_testing = golden_alarms(1501:3000)*

Mini Project 3: Task 2.1

Let: ***H0*** the hypothesis that there is no patient abnormality (no alarm generated).

H1 the hypothesis that there is a patient abnormality (an alarm generated).

a) Use the generated *training* vectors for *HR_AVG*, *PR_AVG*, and *RESP_AVG* variables and *golden_alarms* indicated by the physician, to construct the likelihood matrices for each of the *HR*, *PR*, and *RESP* variables.

   Hint 1: Calculate the probability density for each of the variables under hypothesis *H1* by calculating the frequency of observations in the intervals that an alarm was generated (*golden_training* indicated "1"), similarly for *H0*, you need to look at the intervals that no alarm was generated ("0").

   Hint 2: You can use *tabulate* function in MATLAB, but remember that probability values should be between 0 and 1.

   Hint 3: Remember that in the likelihood matrices, you need to have the same number of columns (possible values of the variable) for both *H1* and *H0*. Find the range of the values that a variable takes by using *min* and *max* commands in MATLAB. Note that the variables might take a value of 0, but MATLAB indices always start from 1.

b) Show your results by plotting the probability density function of each variable in a separate figure. Use the *subplot* function in MATLAB to show the probabilities under each of the hypotheses *H0* and *H1* in one figure.

Mini Project 3: Task 2.2

Similar to what you did in Class Project 4, calculate the joint density functions for each of the following pairs of variables:

- PAIR 1 = (*HR_AVG*, *PR_AVG*)
- PAIR 2 = (*HR_AVG*, *RR_AVG*)

Assume that the variables are independent.

a) For each pair of variables:

- Construct the likelihood matrix for the joint observations.
- Plot the joint density functions for each of the hypotheses *H1* and *H0* (based on the likelihood matrix) in one figure. **Hint:** Use the *mesh* function to draw 3-dimensional plot of joint density for each of the hypotheses (Use *subplot* function)
- Calculate the ML decision rule vector. To be definite, break ties in favor of *H1*.

b) For each pair of variables:

- Compute the joint probability matrix based on the priors of ($\pi_0 = 0.9$, $\pi_1 = 0.1$).
- Plot the joint density functions for each of the hypotheses *H1* and *H0* (based on the joint probability matrix) in one figure. **Hint:** Use *mesh* and *subplot* functions.
- Calculate the MAP decision rule vector. To be definite, break ties in favor of *H1*.

Mini Project 3: Task 3.1

a) Based on the ML and MAP rules found in Task 2.c, implement two different functions P1 (based on ML rule) and P2 (based on MAP rule) for each of the pairs PAIR 1 = (*HR_AVG*, *PR_AVG*) and PAIR 2 = (*HR_AVG*, *RR_AVG*).

Hint: For example, the functions P1 and P2 for PAIR 1 will take as input the joint observations (*HR_AVG*, *PR_AVG*), and use the ML and MAP matrices generated in Task 2.c to generate a vector of "0"s and "1"s, corresponding to the hypotheses *H0* or *H1* declared by the decision rule.

b) Use the *testing* vectors for *HR_AVG*, *PR_AVG*, and *RESP_AVG* (Task 1.b), to evaluate the ML and MAP decision rules computed in the previous part. Use the P1 and P2 functions to generate alarms for each of the *testing* vectors of the pairs 1 and 2.

Hint: You will get two vectors of 1500 columns for each pair, corresponding to the alarms generated by the P1 (ML rule) and P2 (MAP rule) functions.

c) Use the *bar* function in MATLAB to plot the P1 and P2 alarms generated for each of the pairs and the golden alarms (*golden_testing* vector) in one figure.
Hint: Each bar will represent an alarm ("1"). Use *subplot* function.

Mini Project 3: Task 3.2

a) Use the *golden_testing* vector to find the conditional probabilities of false alarms and miss-detections for each of the pairs (*HR_AVG*, *PR_AVG*) and (*HR_AVG*, *RR_AVG*).

Hint: Remember that

- *P(False Alarm) = P(Threshold Function raises an alarm | Physician indicates no abnormality)*
- *P(Miss Detection) = P(Threshold Function raises no alarm | Physician indicates an abnormality)*
- *P(Error) = P(Threshold function raises an alarm AND Physician indicates no abnormality)+*
  P(Threshold function raises no alarm AND Physician indicates an abnormality)

b) Calculate the conditional probabilities of false alarms and miss and the error probability based on the likelihood and joint probability matrices computed for ML and MAP rules in Task 2.2, part c. **Hint:** Similar to Class Project 4.

c) Compare the ML and MAP decision rules for each of the pairs of variables based on the both errors calculated in parts a and b. Which decision rule is better? Based on what criteria? Which pair has a lower error probability?

Provide your insights in a list of 5 bullets.

Project Timeline and Grading

- **TA office hours will be held on Friday, 1:00 – 2:00 PM, in CSL 249, to answer any questions that you may have.**

- **A report (including both the MATLAB code, plots, and results)** must be delivered electronically to: **ece313.B@gmail.com.**
  - Explain all your work and include the code with comments.
  - Show titles, axis labels, and colors to better represent your plots.
  - Write your names and group name on the report.

- **Tasks 0-2: due on Tuesday, November 5, 10:59 AM (before class).**
- **Task 3: due on Friday, November 8, 11:59 PM.**
- **Grading:**
  - **Task 1: 10%**
  - **Task 2: 45%**
  - **Task 3: 45%**