Bernoulli Trials

ECE 313

Probability with Engineering Applications

Lecture 6

Professor Ravi K. Iyer

University of Illinois

Today's Topics

- In-class Project:
  - Series-Parallel Systems Example

- Bernoulli Trials

Bernoulli Trials

- Physical situations of interest:

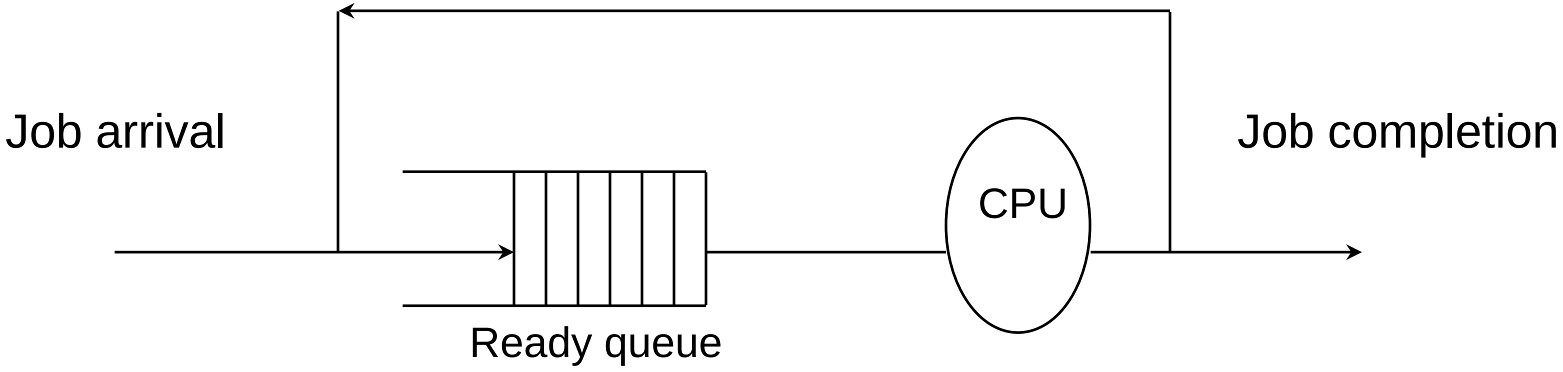
- **1.** Observe n consecutive executions of an **if** statement, with success = "**then** clause is executed" and failure ="**else** clause is executed"

- 2. Examine components produced on an assembly line, with success = "acceptable" and failure = "defective"

- 3. Transmit binary digits through a communication channel, with success = "digit received correctly" and failure = "digit received incorrectly"

Bernoulli Trials (cont.)

4. Consider a time-sharing computer system that allocates a finite quantum (or time slice) to a job scheduled for processor service. Observe n time-slice terminations, with success = “job has completed processing” and failure = “job still requires processing and joins the tail end of the ready queue of processes



Bernoulli Trials (Cont'd)

- Consider a random experiment that has two possible outcomes,. Let the probabilities of the two outcomes be p and q, respectively, with $p + q = 1$.

- Now consider the compound experiment: A sequence of n independent repetitions of this experiment. Such a sequence: is known as a **sequence of Bernoulli trials**.

Bernoulli Trials (cont.)

- Let 0 denote failure and 1 denote success. Let S_n be the sample space of an experiment involving n Bernoulli trials, defined by:
  - $S_1 = \{0, 1\}$, $S_2 = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$
  - $S_n = \{2^n$ n-tuples of 0's and 1's$\}$.

- The probability assignment over the sample space S_1 is already specified: $P(0) = q \geq 0$, $P(1) = p \geq 0$, and $p + q = 1$. We wish to assign probabilities to the points in S_n.

- Let A_i = "Success on trial i" and $\overline{A}_i$ = "Failure on trial i" then $P(A_i) = p$ and $P(\overline{A}_i) = q$.

Bernoulli Trials (cont.)

- Consider s an element of S_n such that s = (1, 1, …, 1, 0, 0, …, 0) [k 1's and (n-k) 0's]. Then the elementary event {s} can be written:

$$\{s\} = A_1 \cap A_2 \cap \ldots \cap A_k \cap \overline{A}_{k+1} \cap \ldots \cap \overline{A}_n$$

$$P(s) = P(A_1 \cap A_2 \cap \ldots \cap A_k \cap \overline{A}_{k+1} \cap \ldots \cap \overline{A}_n)$$

$$= P(A_1)P(A_2)\ldots P(A_k)P)(\overline{A}_{k+1})\ldots P(\overline{A}_n)$$

Bernoulli Trials (cont.)

- Therefore: $P(s) = p^k q^{n-k}$

- Similarly, any sample point with k 1's and (n-k) 0's is assigned probability $p^k q^{n-k}$. Noting that there are $\binom{n}{k}$ such points, the probability of obtaining exactly k successes in n trials is :

$$p(k) = \binom{n}{k} p^k q^{n-k} \qquad k = 0, 1, \ldots, n$$

- Verify that expression for P(s) is a legitimate probability assignment over the sample space Sn since

$$\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = (p+q)^n = 1$$

by the binomial theorem.