Reliability Function and Mean Time to Failure (MTTF) – Class Project 3

ECE 313

Probability with Engineering Applications

Lecture 14

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Today's Topics

- Reliability Function
- Mean Time to Failure (MTTF)

- Class Project 3

- Standby Redundant System

Time to Failure and Reliability Function

- Let *T* denote the time to failure or lifetime of a component in the system, and *f(t)* and *F(t)* denote the probability density function and cumulative distribution function of *T*, respectively.

- *f(t)* represents the momentary probability of failure at time *t*
- The probability that the component will fail **at or before time *t*** is given by: $\mathrm{P}\{\mathrm{T} \le \mathrm{t}\} = \mathrm{F}(\mathrm{t})$

- And the reliability of the component is equal to the probability that it will survive at least until time *t*, given by:

$$R(t) = P\{T > t\} = 1 - F(t)$$

- So we have: $R'(t) = -f(t)$

Mean Time to Failure (MTTF)

- The **expected life** or the **mean time to failure (MTTF)** of the component is given by:

$$E[T] = \int_0^\infty tf(t)dt = -\int_0^\infty tR'(t)dt.$$

- Integrating by parts we obtain:

$$E[T] = \left[-tR(t)\right]_0^\infty + \int_0^\infty R(t)dt.$$

- Now, since *R(t)* approaches zero faster than *t* approaches ∞, we have:

$$\boxed{E[T] = \int_0^\infty R(t)dt = MTTF}$$

Exponentially Distributed Lifetime

- If the component lifetime is exponentially distributed, then:

$$R(t) = e^{-\lambda t}$$

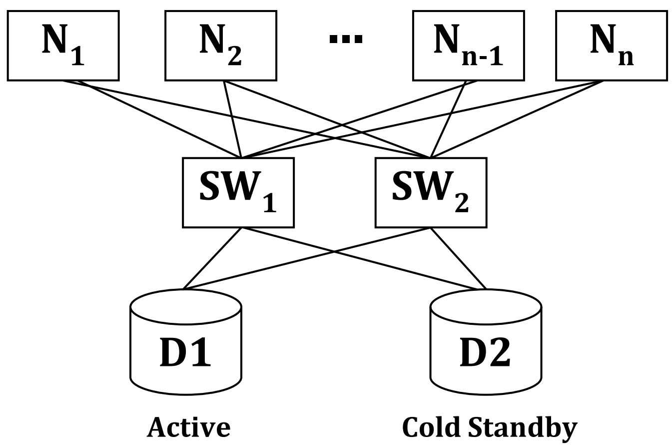
- And:

$$E[T] = \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda}$$

$$Var[T] = \int_0^\infty 2te^{-\lambda t} dt - \frac{1}{\lambda^2} = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Class Project 3

- The following diagram shows the schematic of transaction clearing of a financial exchange system:



- Assume that the clearing system consists of three computing nodes (n = 3), dual redundant switches, and two disk arrays.

Class Project 3 (Cont'd)

- In order for the clearing system to function properly *at least 2 out of 3* computing nodes are required to function (a TMR system) and *at least one* of the redundant switches should work. The disk arrays function as an *active-standby pair*: if the first disk fails, the second disk will be activated, i.e. the second disk is standby idle until the first disk fails.
- Assume that the links and the voter for TMR are perfect, the disk failures are always detected, the recovery to second disk is instantaneous, and if the fist disk fails at time , the second one fails at time .

- The lifetimes (time to failures) of each of the components in the system are exponentially distributed with the following parameters:
  - Computing Nodes: λ_c
  - Switches: λ_s
  - Disk arrays: λ_a

Class Project 3 (Cont'd)

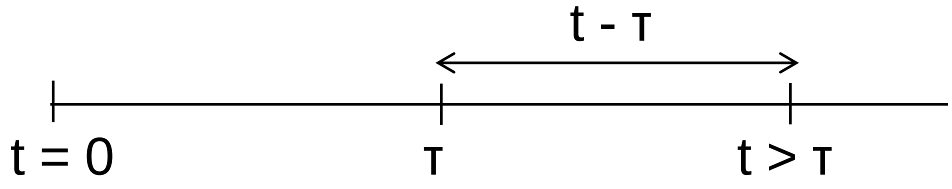
a) Draw the reliability diagram of the whole system.

b) For each of the following subsystems, write the CDF for the time to failure (*T*), and from it derive the reliability function. Use the relation $E[T]=\int_0^\infty R(t)dt$ to calculate the mean time to failure (MTTF):

- Three computing nodes
- Two switches
- Two disk arrays

Standby Redundancy

- A standby system is one in which two components are connected in parallel, but only one component is required to be operative for the system to function properly.
- Initially the power is applied to only one component and the other component is kept in a powered-off state (de-energized).
- When the energized component fails, it is de-energized and removed from operation, and the second component is energized and connected in the former's place.
- If we assume that the first component fails at some time τ, then the second component's lifetime starts at time τ and assuming that it fails at time t, its lifetime will be $t - \tau$:



Standby Redundancy (Cont'd)

- If we assume that the time to failure of the components is exponentially distributed with parameters λ_1 and $\lambda_{2,}$ then the probability density function for the failure of the first component is:

$$f_1(\tau) = \lambda_1 e^{-\lambda_1 t}, \quad 0 < \tau < t$$

- Given that the first component must fail for the lifetime of the second component to start, the density function of the lifetime of the second component is conditional, given by:

$$f_2(t \mid \tau) = \begin{cases} \lambda_2 e^{-\lambda_2 (t-\tau)}, 0 < \tau < t \\ 0 \qquad , \tau > t \end{cases}$$

- Then we define the system failure as a function of *t* and τ, using the definition of conditional probability:

$$\varphi(t,\tau) = f_1(\tau) f_2(t \mid \tau)$$

Standby Redundancy (Cont'd)

- The associated marginal density function of $f(t)$ is:

$$f(t) = \int_0^t \phi(t,\tau)\,d\tau = \int_0^t (\lambda_1 e^{-\lambda_1 \tau})(\lambda_2 e^{-\lambda_2 (t-\tau)})d\tau$$

- So the system failure will be:

$$f(t) = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}(e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

- And the reliability function will be:

$$R(t) = 1 - \int_0^t f(t)\,dt = 1 - \int_0^t \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}(e^{-\lambda_2 t} - e^{-\lambda_1 t})dt$$

$$= \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_1 - \lambda_2}$$

Standby Redundancy (Cont'd)