ECE 313: Problem Set 13
Review Problems

Due: Not due; no credit involved in this homework
Reading: *ECE 313 Course Notes.*

1. []
   $\mathbb{X}$ and $\mathbb{Y}$ are jointly continuous random variables with joint pdf given by

$$f_{\mathbb{X},\mathbb{Y}}(u,v) = \begin{cases} 2, & \text{if } u \geq 0,\ v \geq 0,\ u+v \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find the pdf $f_{\mathbb{Z}}(\alpha)$ of the random variable $\mathbb{Z} = \mathbb{Y}/\mathbb{X}$. To obtain full credit, you must specify the value of $f_{\mathbb{Z}}(\alpha)$ for all α, $-\infty < \alpha < \infty$.

2. []
   The joint pdf of the random variables $\mathbb{X}$ and $\mathbb{Y}$ has constant value 1 on the triangular region with vertices at $(-1,0)$, $(0,1)$, and $(1,0)$.

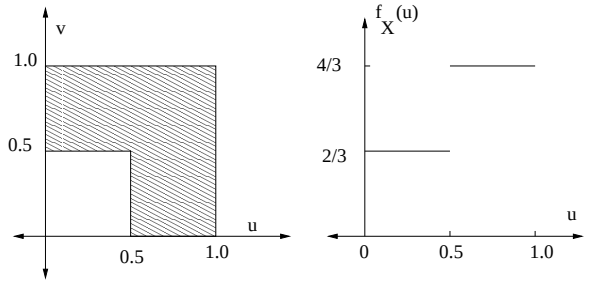
   (a) Find the value of $\mathsf{E}[\mathbb{X}]$ and $\mathsf{E}[\mathbb{Y}]$.

   (b) Find $f_{\mathbb{Y}|\mathbb{X}}(v \mid \alpha)$, the conditional pdf of $\mathbb{Y}$ given that $\mathbb{X} = \alpha$.

   (c) Find, as a function of u, a graph of the *minimum mean-square error estimator* of $\mathbb{Y}$ given that value of $\mathbb{X}$ is u, for u in the range $u \in (-1,1)$.

   (d) Find the **linear** *minimum mean-square error estimator* of $\mathbb{Y}$ given that value of $\mathbb{X}$ is u, where $u \in (-1,1)$.

3. []
   Let (X,Y) be uniformly distributed over the region shown. Also shown, for your convenience, is the pdf of X, which is the same as the pdf of Y.



   (a) Find the function $g(X)$ which gives the minimum mean square error (MMSE) estimator of Y given X.

   (b) Find the (average) mean square error for the MMSE estimator of Y given X.

   (c) Find $E[X]$.

   (d) Find $E[XY]$.

   (e) Find the linear function $L(u)$, of the form $au + b$, such that $L(X)$ is the linear estimator of Y based on X with the minimum MSE.