University of Illinois **Fall 2011**

ECE 313: Problem Set 11
Distribution of sums of random variables and additional examples using joint distributions

Due:	Wednesday, November 9, at 4 p.m.
Reading:	*ECE 313 Notes* Sections 4.5&4.6
Reminder:	Hour Exam II will be held Monday November 14, 6.40 pm to 8 pm Section D (11 am section) Room 112 Chem Annex, all other sections: Room 314 Altgeld Hall One two-sided 8.5"×11" sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring a picture ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-11, with an emphasis on problem sets 7-11.

1. **[Sums of two random variables]**
   Let X and Y be continuous random variables with joint pdf
   $$f_{X,Y}(u,v) = \begin{cases} \frac{x}{5} + cy & 0 < x < 1, 1 < y < 5 \\ 0 & \text{else,} \end{cases}$$
   where c is a constant.
   (a) Find the value of the constant c.
   (b) Are X and Y independent?
   (c) Find $P\{X + Y > 3\}$

2. **[Sums of random variables]**
   Two random variables X and Y have the following joint pdf:
   $$f_{X,Y}(u,v) = \begin{cases} 2\exp(-(u+v)) & 0 < u < v < \infty \\ 0 & \text{else.} \end{cases}$$
   A new random variable Z is defined as: Z=X+Y. Find the CDF and pdf of Z.

3. **[Joint densities]**
   Consider two components and three types of shocks. A type 1 shock causes component 1 to fail, a type 2 shock causes component 2 to fail, and a type 3 shock causes both components 1 and 2 to fail. The time until shocks 1, 2 and 3 occur are independent exponential random variables with rates λ_1, λ_2, and λ_3, respectively. Let X_i denote the time at which component i fails, for $i = 1, 2$.
   (a) Find $P\{X_1 > s, X_2 > t\}$.
   (b) Find the CDF of X_1, $F_{X_1}(s)$.

4. **[Joint densities and functions of two random variables]**
   Let X and Y have joint pdf
   $$f_{X,Y}(u,v) = \begin{cases} A\left(1 - \sqrt{u^2+v^2}\right) & u^2+v^2 < 1 \\ 0 & \text{else.} \end{cases}$$
   Hint: Use of polar coordinates is useful for all parts of this problem.
   (a) Find the value of A.
   (b) Let $Z = X^2 + Y^2$. Find the pdf of the random variable Z.
   (c) Find $E[Z^5]$ using LOTUS for joint pdfs: $E[g(X,Y)] = \int\int_{\mathbb{R}^2} g(u,v) f_{X,Y}(u,v)dudv$.

5. **[Joint densities and functions of two random variables]**
   Let X and Y be two independent random random variables, each uniformly distributed on the interval (0,1). Let $Z = \frac{1}{X+Y}$ and find the pdf $f_Z(a)$.