

ECE 313: Problem Set 10

Joint Distributions, Independence

Due: Wednesday, November 2 at 4 p.m.

Reading: *ECE 313 Course Notes*, Sections 4.1-4.4.

1. **[A joint pmf]**

The joint pmf $p_{X,Y}(u, v)$ of X and Y is shown in the table below.

	u=0	u=1	u=2	u=3
v=4	0	0.1	0.1	0.2
v=5	0.2	0	0	0
v=6	0	0.2	0.1	0.1

- (a) Find the marginal pmfs $p_X(u)$ and $p_Y(v)$.
- (b) Let $Z = X + Y$. Find p_Z , the pmf of Z .
- (c) Are X and Y independent random variables? Justify your answer.
- (d) Find $p_{Y|X}(v|3)$ for all v and find $E[Y|X = 3]$.

2. **[A joint distribution]**

Suppose X and Y are jointly continuous with joint pdf

$$f_{X,Y}(u, v) = \begin{cases} ve^{-(1+u)v} & u \geq 0, v \geq 0 \\ 0 & \text{else.} \end{cases}$$

- (a) Find the marginal pdfs, f_X and f_Y . Note that these functions are defined on the entire real line. (Hint: To find the marginal of X it might help to review the gamma density.)
- (b) Find the conditional pdfs, $f_{Y|X}$ and $f_{X|Y}$. Be sure to indicate where these functions are well defined, and where they are zero, as well as giving the nonzero values.
- (c) Find the joint CDF, $F_{X,Y}(u_o, v_o)$.
- (d) Are X and Y independent? Justify your answer.

3. **[Recognizing independence]**

Decide whether X and Y are independent for each of the following three joint pdfs. If they are independent, identify the marginal pdfs f_X and f_Y . If they are not, give a reason why.

- (a) $f_{X,Y}(u, v) = \begin{cases} \frac{4}{\pi} e^{-(u^2+v^2)} & u, v \geq 0 \\ 0 & \text{else.} \end{cases}$
- (b) $f_{X,Y}(u, v) = \begin{cases} -\frac{\ln(u)v^2}{21} & 0 \leq u \leq 1, 1 \leq v \leq 4 \\ 0 & \text{else.} \end{cases}$
- (c) $f_{X,Y}(u, v) = \begin{cases} \frac{(96)u^2v^2}{\pi} & u^2 + v^2 \leq 1 \\ 0 & \text{else.} \end{cases}$

4. **[Joint pmfs]**

The joint pmf $p_{X,Y}(u, v)$ of X and Y is shown in the table below.

- (a) Find the marginal pmfs $p_X(u)$ and $p_Y(v)$.
- (b) Are X and Y independent random variables?
- (c) Find $P\{X \leq Y\}$ and $P\{X + Y \leq 4\}$.

$u \rightarrow$		0	1	3	5
$v \downarrow$					
	4	0	1/12	1/6	1/12
	3	1/6	1/12	0	1/12
	-1	1/12	1/6	1/12	0

(d) Find $p_{\mathbb{X}|\mathbb{Y}}(u | 3)$, $E[\mathbb{X} | \mathbb{Y} = 3]$ and $\text{var}(\mathbb{X} | \mathbb{Y} = 3)$.

5. \square

The joint pdf of \mathbb{X} and \mathbb{Y} is given by

$$f(u, v) = c(v^2 - u^2) \exp(-v), \quad -v \leq u \leq v, \quad 0 < v < \infty.$$

(a) Find c .

(b) Find the marginal densities of \mathbb{X} and \mathbb{Y} .

(c) Find $E[\mathbb{X}]$.

6. **[Drill problem]**

The jointly continuous random variables \mathbb{X} and \mathbb{Y} have joint pdf

$$f_{\mathbb{X},\mathbb{Y}}(u, v) = \begin{cases} 2 \exp(-u - v), & 0 < u < v < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Sketch the u - v plane and indicate on it the region over which $f_{\mathbb{X},\mathbb{Y}}(u, v)$ is nonzero.

(b) Find the marginal pdfs of \mathbb{X} and \mathbb{Y} .

(c) Are the random variables \mathbb{X} and \mathbb{Y} independent?

(d) Find $P\{\mathbb{Y} > 3\mathbb{X}\}$.