University of Illinois **Fall 2011**

ECE 313: Problem Set 8
Poisson process, Linear scaling, Gaussian distribution

Due: Wednesday October 19 at 4 p.m.
Reading: *313 Course Notes* Sections 3.5–3.6

1. **[Poisson process]**
A certain application in a cloud computing system is accessed on average by 15 customers per minute. We model the accesses as a Poisson process. Find the probability that in a one minute period, three customers access the application in the first ten seconds and two customers access the application in the last fifteen seconds. (Any number could access the system in between these two time intervals.)

2. **[Poisson process probabilities]**
Consider a Poisson process with arrival rate $\lambda > 0$.
   (a) Find the probability there is exactly one arrival in each of the intervals (0,1], (1,2], and (2,3].
   (b) Find the probability that there are two arrivals in the interval $(0, 2]$ and two arrivals in the interval $(1, 3]$. (Hint: The occurrence of the event in (a) implies the occurrence of the event in (b). What other ways can the event in (b) occur?)
   (c) Find the probability that there are two arrivals in the interval (1,2], given that there are two arrivals in the interval (0,2] and two arrivals in the the interval (1,3].

3. **[Gaussian Distribution]**
The random variable X has a $\mathcal{N}(-4, 9)$ distribution. Determine the following probabilities using the normal tables in Section 6.3 of the notes. If a value is not in the table, use the closest value.
   (a) $P\{X = 0\}$
   (b) $P\{|X + 4| \geq 2\}$
   (c) $P\{0 < X < 2\}$
   (d) $P\{X^2 < 9\}$

4. **[Enhancing the Yield of Cell Phones]**
A company manufactures cell phones. A specific resistor in the cell phone electronics has a value R that fluctuates from one phone to the next with a mean value of $1k\Omega$ and variance of $10^4\Omega^2$. A fixed current $I = 1mA$ flows from terminal A to terminal B of the resistor creating a potential difference $V_A - V_B = IR$ across it, where V_A and V_B are the potentials (voltages) at terminals A and B, respectively. The electronic circuit in the cell phone is such that $V_B = 1V$.
   (a) Derive the mean and variance of potential V_A.
   (b) Resistor value R is known to be Gaussian distributed. The cell phone electronics is known to fail if $V_A > 2.05V$ or $V_A < 1.95V$. Determine Y, the percentage of cell phones manufactured that will actually work.
   (c) The low yield in Part (b) has the company executives worried that they may go out of business soon. They charge the engineering team to bring up the yield to at least 90%. The engineering team decides to install a precision resistor, i.e., a resistor with a smaller tolerance/variation. What should the variance of this precision resistor be so that the yield is 90%?

5. **[DeMoivre-Laplace approximation to central term of binomial distribution]**
Let n be a positive *even* integer, and let $\mathbb{X}$ be a binomial random variable with parameters $(n, 0.5)$. This problem focuses on $P\{\mathbb{X} = \frac{n}{2}\}$. The continuity correction for approximating the binomial distribution by the normal distribution begins by writing this same probability as $P\{\frac{n-1}{2} \leq \mathbb{X} \leq \frac{n+1}{2}\}$.

(a) Using the continuity correction, find the normal approximation to $P\{\mathbb{X} = \frac{n}{2}\}$. Your answer should involve n and the standard normal CDF $\Phi(x)$.

(b) Find the constant c such that $\sqrt{n}P\{\mathbb{X} = \frac{n}{2}\} \to c$ as $n \to \infty$, assuming you can replace $P\{\mathbb{X} = \frac{n}{2}\}$ by its normal approximation found in part (a). This suggests that $P\{\mathbb{X} = \frac{n}{2}\} \approx \frac{c}{\sqrt{n}}$ for large n. (Hint: Since $\Phi(x)$ is differentiable for all x, then $\frac{\Phi(h)-\Phi(0)}{h} \to \frac{d}{dx}\Phi(x)\big|_{x=0} = \Phi'(0)$ as $h \to 0$.)

(c) For $n = 30$, compute up to six decimal place the exact value of $P\{\mathbb{X} = \frac{n}{2}\}$, the approximation found in part (a), and the approximation found in part (b).