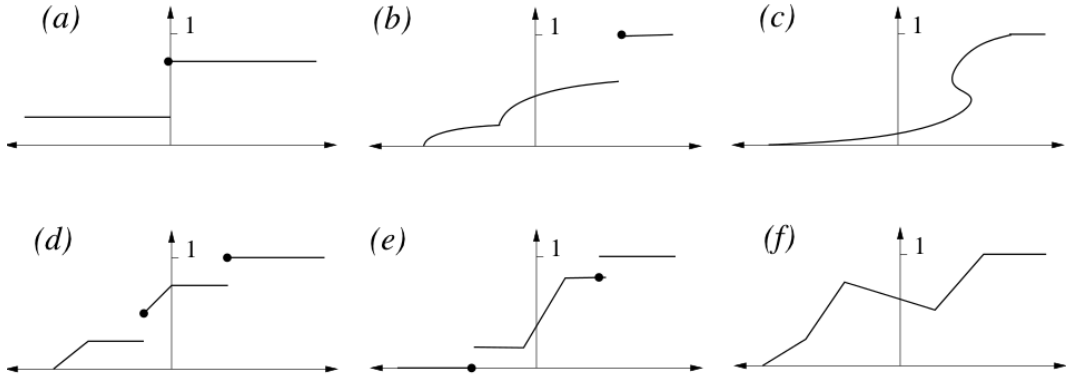
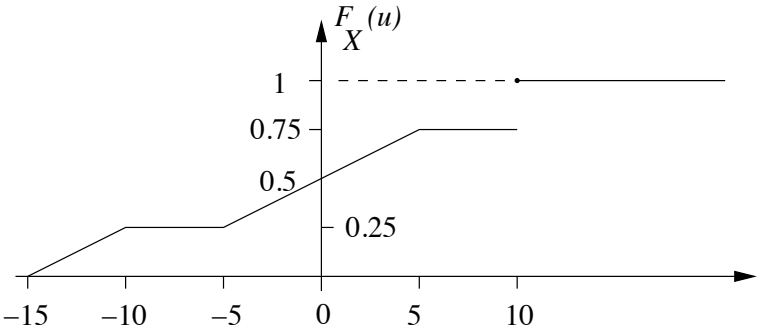
ECE 313: Problem Set 7
CDFs, continuous-type random variables, uniform and exponential distributions

Due: **Friday** October 14 at 4 p.m.
Reading: *313 Course Notes* Sections 3.1–3.4

1. **[CDFs]**
Which of the six plots below show valid CDFs? For each one that is not valid, state a property of CDFs that is violated.



2. **[CDFs]**
Let X be a random variable with the CDF shown.



Compute the following probabilities:

(a) $P\{X = 0\}$
(b) $P\{X = 10\}$
(c) $P\{X < 10\}$
(d) $P\{X \geq -6\}$
(e) $P\{|X| < -10\}$
(f) $P\{X^2 \leq 16\}$

3. **[Continuous-type random variables]**
In each case given below, determine whether $f(x)$ is a valid pdf or not. For those which are not valid pdfs, state at least one property of pdfs which is not satisfied and also indicate if there exists a constant c such that $cf(x)$ is a valid probability density, and if so, find the value of c. The function $f(x)$ is equal to zero outside of the ranges indicated.

(a) $f(x) = x^{-1}$ for $1 \leq x < \infty$.

(b) $f(x) = \ln(x)$ for $0 < x < 1$.

(c) $f(x) = \frac{2}{3}(x-1)$ for $0 < x < 3$.

(d) $f(x) = |x|$ for $|x| < 1$.

(e) $f(x) = e^{-2x}$ for $0 < x < \infty$.

(f) $f(x) = x^2 e^{-x}$ for $0 \leq x < \infty$.

4. **[Continuous-type random variables]**
Let X be a random variable with mean $5/2$, variance $1/12$ and pdf given by:
$f_X(u) = \begin{cases} a + bu + cu^2, & 2 \leq u \leq 3 \\ 0, & else. \end{cases}$

(a) Find a, b and c.

(b) Find $P\{1 \leq X < 4\}$.

(c) Find $P\{X > 5/2\}$.

5. **[Uniform and exponential distributions]**
Let X be a continuous random variable uniformly distributed over the interval $(1, 10)$ and let Y be continuous random variable exponentially distributed with parameter $\lambda = 2$. What is the probability that

(a) X is equal to 5?

(b) Y is larger than 3 given that Y is larger than 1?

(c) the integer part of X will be a 1?

(d) the first decimal digit of Y will be a 2?

(e) the first decimal digit of the square root of X will be a 3?