ECE 313: Problem Set 2
Discrete random variables

Due: Wednesday September 7, at 4 p.m..
Reading: ECE 313 notes, Chapter 1 and Sections 2.1-2.2.

1. **[Maximum and minimum values for probabilities]**
   A, B and C are events with probabilities 0.6, 0.2, and 0.7 respectively.
   (a) What are the largest possible values of $P(A \cup B)$ and $P(A \cup C)$?
   (b) What are the smallest possible values of $P(A \cup B)$ and $P(A \cup C)$?
   (c) What are the largest possible values of $P(A \cap B)$ and $P(A \cap C)$?
   (d) What are the smallest possible values of $P(A \cap B)$ and $P(A \cap C)$?

2. **[Mean and standard deviation]**
   Suppose three fair dice are rolled independently, so the sample space is $\Omega = \{(i,j,k) : 1 \le i,j,\ell \le 6\}$ and all outcomes are equally likely. Let X be the number showing on the first die, $X(i,j,\ell) = i$, and let Y be the random variable defined by $Y(i,j,k) = \min\{i,j,\ell\}$.
   (a) Derive the pmf of X and sketch it.
   (b) Find the mean $E[X]$ and standard deviation, σ_X, of X. Correct numerical answers are fine, but show your work.
   (c) Derive the pmf of Y and sketch it.
   (d) Find the mean $E[Y]$ and standard deviation, σ_Y, of Y.
   (e) Which is larger, σ_X or σ_Y? Is that consistent with your sketches of the pmfs?

3. **[Countably infinite sample space]**
   A fair die is rolled once. Let $f_1 \in \{1,2,3,4,5,6\}$ denote the outcome. The die is then rolled repeatedly till an outcome f_2 that is *different* from f_1 occurs.
   (a) Find the probability that f_1 is even.
   (b) Find the probability that both f_1 and f_2 are even.
   (c) Find the probability that $f_1 + f_2 \le 7$.

4. **[Mean, Variance, LOTUS]**
   Consider a random variable X.
   (a) Is it possible for the mean of X to be 10 and standard deviation 0? If so, construct a pmf for X that corresponds to these values. If not, prove the impossibility.
   (b) Is it possible for the mean of X to be 0 and standard deviation 10? If so, construct a pmf for X that corresponds to these values. If not, prove the impossibility.
   (c) Suppose the mean of X is μ and the standard deviation is σ. Find a formula for the mean of the random variable $Y = 6 - 5X + X^2$ in terms of μ and σ.

5. **[Defining Events]**
   Express each of the following events in terms of the events A, B, and C, and the operations of complementation, union, and intersection.
   (a) at least one of the events A, B, C occurs;
   (b) none of the events A, B, C occurs;

(c) all three events A, B, C occur;

(d) exactly one of the events A, B, C occurs;

(e) at most one of the events A, B, C occurs;

(f) events A and B occur, but not C;

(g) either event A occurs, or if not then B also does not occur.