ECE 313: Problem Set 1
Axioms of probability and calculating the sizes of sets

Due: Wednesday, August 31 at 4 p.m.
Reading: *ECE 313 Course Notes,* Sections 1.1–1.4.

1. **[Defining a set of outcomes]**
   Suppose four teams, numbered one through four, play a round-robin tournament: every pair of teams play a game. Two teams play each game and one of them wins; ties do not occur.

   (a) Define a set Ω so that the elements of Ω correspond to the possible scenarios of the tournament. An element of Ω should specify the entire sequence of outcomes of the games. Explain how the elements of your set correspond to the possible scenarios. (This is an exercise in coming up with good notation.)

   (b) How many possible scenarios are there?

2. **[Possible probability assignments]**
   Suppose A and B are events for some probability space such that $P(AB) = 0.2$ and $P(A \cup B) = 0.5$. Find the set of possible values of the pair $(P(A), P(B))$ and sketch this set, as a subset of the plane. Hint: It might help to try filling in the probabilities in a Karnaugh map.

3. **[Grouping students into teams]**
   Suppose twelve students in a class are to be grouped into teams.

   (a) If each team has three students, how many ways are there to form teams? (The ordering of students within teams does not matter, and the ordering of the teams does not matter.)

   (b) If each team has either two or three students, how many ways are there to form teams?

4. **[A Karnaugh map for three events]**
   Let an experiment consist of rolling two fair dice, and define the following three events about the numbers showing: $A =$"sum is odd," $B =$"sum is a multiple of three," and $C =$"the number showing on the first die is (strictly) less than the number showing on the second die."

   (a) Display the outcomes in a three-event Karnaugh map, as in Example 1.4.2.

   (b) Find $P((A \cup B)C)$.

5. **[Cards]**
   Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) *E1* is the event that all five cards are one of A, K, Q, J (though not necessarily of the same suit). Find $P(E1)$.

   (b) *E2* is the event that either all the five cards are from the same suit or four of the five cards are A, K, Q, J from a common suit. Find $P(E2)$.