

## ECE 313: Hour Exam II

1. [10 points] Buses arrive at a bus station in a Poisson process with an average of 2 buses per hour.
  - (a) [5 points] Let  $T_1$  be the time of arrival of the first bus and  $T_2$  be the time of arrival of the second bus, as measured in minutes. What is the pdf of the random variable  $\mathbb{T} = T_2 - T_1$ ?
  - (b) [5 points] You reach the bus station at 5pm. The next bus arrives at time 5pm +  $\mathbb{X}$  minutes. What is the expected value of the random variable  $\mathbb{X}$ ?

2. **[25 points]** Let  $\mathbb{X}$  be a continuous random variable with variance 100. The pdf of  $\mathbb{X}$  is symmetric around the origin, i.e., for all  $u$ , we have  $f_{\mathbb{X}}(u) = f_{\mathbb{X}}(-u)$ .

(a) **[5 points]** Find the mean of  $\mathbb{X}$ .

(b) **[10 points]** Let  $\mathbb{Y} = 2\mathbb{X} + 10$ . Find the mean and variance of  $\mathbb{Y}$ .

(c) **[5 points]** Let  $\mathbb{Z} = \mathbb{X}^2$ . Find the mean of  $\mathbb{Z}$ .

(d) **[5 points]** Let  $\mathbb{W} = |\mathbb{X}|$ . It is claimed that the variance of  $\mathbb{W}$  is more than 100. Mark TRUE or FALSE by checking one box below. You need not justify your answer.

TRUE    FALSE

3. [15 points] Consider the following binary hypothesis testing problem.

If hypothesis  $H_0$  is true, the continuous random variable  $\mathbb{X}$  is uniformly distributed on  $(-1, 1)$ ,

while if hypothesis  $H_1$  is true, the pdf of  $\mathbb{X}$  is  $f_1(u) = \begin{cases} C(1 - |u|), & |u| < 1, \\ 0, & \text{otherwise.} \end{cases}$

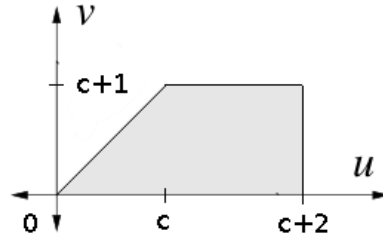
(a) [5 points] Find the value of  $C$ .

(b) [10 points] Find the decision region  $\Gamma_0$  for the *maximum-likelihood* decision rule. Remember that  $\Gamma_0$  is the set of all real numbers such that if  $\mathbb{X} \in \Gamma_0$ , the decision is that  $H_0$  is the true hypothesis.

4. **[10 points]** Consider a uniform random variable  $\mathbb{X}$  over the interval  $[-1, 1]$ . Find the pdf of  $\mathbb{Y} = \mathbb{X}^2$ . Take care to specify the value of the pdf  $f_{\mathbb{Y}}(v)$  at all values  $v$ .

5. **[20 points]** The random variable  $X$  has a  $\mathcal{N}(10, 4)$  distribution. Determine the following probabilities using the normal tables attached. If a value is not in the table, use the closest value.
- (a) **[10 points]**  $P\{X > 10\}$ .
  - (b) **[10 points]**  $P\{X \geq 12\}$ .

6. [20 points] Let  $X$  and  $Y$  be two continuous-type random variables with joint pdf  $f_{X,Y}(u,v)$  having support in the shaded region below, where  $c \geq 0$  is a constant. The joint pdf is *uniform*, i.e.,  $f_{X,Y}(u,v) = A$  for all  $(u,v)$  in the shaded region for some  $A > 0$  and zero otherwise.



- (a) [10 points] Find the value of  $A$ . Your answer should be a function of  $c$ .  
(b) [10 points] Find the value of the constant  $c$  such that  $X$  and  $Y$  are independent.

**6.3 Normal tables**

Tables 6.1 and 6.2 below were computed using Abramowitz and Stegun, *Handbook of Mathematical Functions*, Formula 7.1.26, which has maximum error at most  $1.5 \times 10^{-7}$ .

Table 6.1:  $\Phi$  function, the area under the standard normal pdf to the left of  $x$ .

<b>x</b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
<b>0.0</b>	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
<b>0.1</b>	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
<b>0.2</b>	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
<b>0.3</b>	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
<b>0.4</b>	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
<b>0.5</b>	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
<b>0.6</b>	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
<b>0.7</b>	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
<b>0.8</b>	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
<b>0.9</b>	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
<b>1.0</b>	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
<b>1.1</b>	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
<b>1.2</b>	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
<b>1.3</b>	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
<b>1.4</b>	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
<b>1.5</b>	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
<b>1.6</b>	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
<b>1.7</b>	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
<b>1.8</b>	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
<b>1.9</b>	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
<b>2.0</b>	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
<b>2.1</b>	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
<b>2.2</b>	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
<b>2.3</b>	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
<b>2.4</b>	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
<b>2.5</b>	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
<b>2.6</b>	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
<b>2.7</b>	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
<b>2.8</b>	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
<b>2.9</b>	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
<b>3.0</b>	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Table 6.2:  $Q$  function, the area under the standard normal pdf to the right of  $x$ .

$x$	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
<b>0.0</b>	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
<b>0.1</b>	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
<b>0.2</b>	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
<b>0.3</b>	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
<b>0.4</b>	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
<b>0.5</b>	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
<b>0.6</b>	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
<b>0.7</b>	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
<b>0.8</b>	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
<b>0.9</b>	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
<b>1.0</b>	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
<b>1.1</b>	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
<b>1.2</b>	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
<b>1.3</b>	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
<b>1.4</b>	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
<b>1.5</b>	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
<b>1.6</b>	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
<b>1.7</b>	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
<b>1.8</b>	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
<b>1.9</b>	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
<b>2.0</b>	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
<b>2.1</b>	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
<b>2.2</b>	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
<b>2.3</b>	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
<b>2.4</b>	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
<b>2.5</b>	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
<b>2.6</b>	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
<b>2.7</b>	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
<b>2.8</b>	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
<b>2.9</b>	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
<b>3.0</b>	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

$x$	<b>0.0</b>	<b>0.2</b>	<b>0.4</b>	<b>0.6</b>	<b>0.8</b>
<b>0.0</b>	0.5000000	0.4207403	0.3445783	0.2742531	0.2118553
<b>1.0</b>	0.1586553	0.1150697	0.0807567	0.0547993	0.0359303
<b>2.0</b>	0.0227501	0.0139034	0.0081975	0.0046612	0.0025552
<b>3.0</b>	0.0013500	0.0006872	0.0003370	0.0001591	0.0000724
<b>4.0</b>	0.0000317	0.0000134	0.0000054	0.0000021	0.0000008