ECE 313: Problem Set 8
Linear Scaling, Gaussian Distribution, ML Parameter Estimation

Due:	Wednesday, October 20 at 4 p.m.
Reading:	*ECE 313 Notes* Sections 3.6 & 3.7

1. [**Gaussian Distribution**]
   The random variable X has a $\mathcal{N}(-4, 9)$ distribution. Determine the following probabilities:

   (a) $P\{X = 0\}$

   (b) $P\{|X + 4| \geq 2\}$

   (c) $P\{0 < X < 2\}$

   (d) $P\{X^2 < 9\}$

2. [**Enhancing the Yield of Cell Phones**]
   A company manufactures cell phones. A specific resistor in the cell phone electronics has a value R that fluctuates from one phone to the next with a mean value of $1k\Omega$ and variance of $10^4\Omega^2$. A fixed current $I = 1mA$ flows from terminal A to terminal B of the resistor creating a potential difference $V_A - V_B = IR$ across it, where V_A and V_B are the potentials (voltages) at terminals A and B, respectively. The electronic circuit in the cell phone is such that $V_B = 1V$.

   (a) Derive the mean and variance of potential V_A.

   (b) Resistor value R is known to be Gaussian distributed. The cell phone electronics is known to fail if $V_A > 2.05V$ or $V_A < 1.95V$. Determine Y, the percentage of cell phones manufactured that will actually work.

   (c) The low yield in Part (b) has the company executives worried that they may go out of business soon. They charge the engineering team to bring up the yield to at least 90%. The engineering team decides to install a precision resistor, i.e., a resistor with a smaller tolerance/variation. What should the variance of this precision resistor be so that the yield is 90%?

3. [**Approximations to a Binomial Distribution**]
   A communication receiver recovers a block of $n = 10^5$ bits. It is known that each bit in the block can be in error with probability 10^{-4}, independently of whether other bits are in error.

   (a) Write down an exact expression for the probability of observing $k = 15$ errors in the block. A numerical value isn't required to be calculated.

   (b) Determine an approximate value of $P\{X = 15\}$ via the Gaussian approximation with continuity correction.

   (c) Solve part (b) using the Poisson approximation of a binomial distribution.

4. [**ML Parameter Estimation**]
   Calls arrive in a call center according to a Poisson process with arrival rate λ (calls/minute). Derive the maximum likelihood estimate $\widehat{\lambda}_{ML}$ if k calls are received in a T minute interval.