University of Illinois **Fall 2010**

ECE 313: Problem Set 1-11
Randomly selected problems for ECE 313 review session Friday, 11/12/10

Note: This is a compilation of randomly selected problems. There is one problem each from problem sets 7-11, and four problems selected at random from among those on Exam II for the last six semesters of 313. These problems were selected independently of Exam II and may or may not be similar to problems on the exam. **They definitely do not cover all the material that could be covered on the exam. The purpose of the document is solely to aid in the review session led by the instructors on Friday, November 12, during class. The complete list of topics the exam is selected from is given by the table of contents of the notes, from Sections 3.1 to 4.6.**

Reminder: Hour Exam II will be held Monday November 15, 7:00 p.m. – 8:00 p.m.
Section C (10 am section) Room 124 Burrill Hall, all other sections: Room 100 Noyes Laboratory
One two-sided 8.5”×11” sheet of notes allowed, with font size no smaller than 10 pt or equivalent handwriting. Bring a picture ID. The exam will cover the reading assignments, lectures, and problems associated with problem sets 1-11, with an emphasis on problem sets 7-11.

1. **[Uniform and exponential distribution, II-PS7, number 4]**
A factory produced two equal size batches of radios. All the radios look alike, but the lifetime of a radio in the first batch is uniformly distributed from zero to two years, while the lifetime of a radio in the second batch is exponentially distributed with parameter $\lambda = 0.1(\text{years})^{-1}$.
   (a) Suppose Alicia bought a radio and after five years it is still working. What is the conditional probability it will still work for at least three more years?
   (b) Suppose Venkatesh bought a radio and after one year it is still working. What is the conditional probability it will work for at least three more years?

2. **[Gaussian Distribution-ps8, number 1]**
The random variable X has a $\mathcal{N}(-4, 9)$ distribution. Determine the following probabilities:
   (a) $P\{X = 0\}$
   (b) $P\{|X + 4| \geq 2\}$
   (c) $P\{0 < X < 2\}$
   (d) $P\{X^2 < 9\}$

3. **[Generation of random variables with specified probability density function PS 9, number 3]**
Find a function g so that, if U is uniformly distributed over the interval $[0, 1]$, and $X = g(U)$, then X has the pdf:
$$f_X(v) = \begin{cases} 2v & \text{if } 0 \leq u \leq 1 \\ 0 & \text{else.} \end{cases}$$
(Hint: Begin by finding the cumulative distribution function F_X.)

4. **[Recognizing independence PS10-number 4]**
Decide whether X and Y are independent for each of the following three joint pdfs. If they are independent, identify the marginal pdfs f_X and f_Y. If they are not, give a reason why.
   (a) $f_{X,Y}(u, v) = \begin{cases} \frac{4}{\pi} e^{-(u^2+v^2)} & u, v \geq 0 \\ 0 & \text{else.} \end{cases}$
   (b) $f_{X,Y}(u, v) = \begin{cases} -\frac{ln(u)v^2}{21} & 0 \leq u \leq 1, 1 \leq v \leq 4 \\ 0 & \text{else.} \end{cases}$

(c) $f_{X,Y}(u,v) = \begin{cases} \frac{(96)u^2v^2}{\pi} & u^2+v^2 \leq 1 \\ 0 & \text{else.} \end{cases}$

5. **[Joint densities, PS 11, number 1]**
X and Y are two random variables with the following joint pdf:

$$f_{X,Y}(u,v) = \begin{cases} A(1-|u-v|) & 0 < u < 1, 0 < v < 1 \\ 0 & else \end{cases}$$

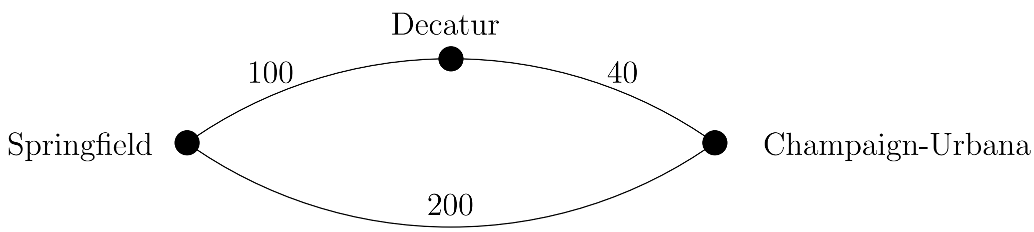
(a) Find A.

(b) Find marginal pdfs for X and Y.

(c) Find $P\{X > Y\}$.

(d) Find $P(X + Y < 1 | X > 1/2)$.

6. **[15 points]** (Fall 2009, Exam 2, number 1) Messages can be transmitted between Champaign-Urbana and Springfield either directly or via Decatur on communication links whose capacities are indicated on the diagram shown below. Assume that the three links fail independently with equal probability $\frac{1}{2}$. Let X denote the communication capacity between Champaign-Urbana and Springfield.



(a) **[5 points]** What values can X take on?

(b) **[10 points]** Find the expected value of X.

7. **[Fall 2008 Exam 2, number 3]**
Let X denote a Gaussian random variable with mean 10 and variance 100. Find the value of $P(X > 71.7 | X > 29.6)$. A table of $\Phi(x)$, the CDF of the standard Gaussian distribution, is attached to this examination.

8. **[Spring 2008, Exam II, problem 2]**

(a) W denotes a *uniform* random variable with mean 1 and variance 3. Find $P\{W < 0\}$.

(b) Suppose X is an exponential random variable with parameter λ. Calculate the pdf of the random variable $Y = \sqrt{2\lambda X}$.

(c) Suppose that Z is a standard Gaussian random variable. Find $E[|Z|]$.

9. **[Fall 2009 Exam 2, number 5 points]** X denotes a *uniform* random variable with mean 1 and variance 3.

(a) **[10 points]** Find $P\{X < 0\}$.

(b) **[15 points]** Find the pdf of $Y = |X|$. In order to receive full credit, you must specify the value of $f_Y(v)$ for all $v, -\infty < v < \infty$.