University of Illinois Fall 2010

ECE 313: Hour Exam I

Monday October 11, 2010
7:00 p.m. — 8:00 p.m.
Room 141 Wohlers Hall & Room 114 David Kinley Hall

1. (a) $P(AB^cC^c) = 0.01$.
   (b) $P(AB \cup BC \cup AC) = 0.12$.
   (c) $P(A \cup B \cup C)^c = 0.68$.
   (d) $P(AB \cup BC) = 0.11$.
   (e) $P(ABC^c \cup A^cBC) = 0.1$.

2. (a) Let T represent the event that the exam committee is tough.

$$P(T) = \frac{6\binom{4}{2} + \binom{4}{3}}{\binom{10}{3}} = \frac{6 \cdot 6 + 4}{\frac{10 \cdot 9 \cdot 8}{3 \cdot 2}} = \frac{40}{120} = \frac{1}{3}$$

   (b) Let A denote the event a petition is approved. By the law of total probability,

$$P(A) = P(A|T)P(T) + P(A|T^c)P(T^c) = (0.5)(1/3) + (0.8)(2/3) = 0.7$$

   (c) The number of tries until success has the geometric distribution with parameter $p = 3/4$, so the expected number of tries is $1/p = 4/3$.
   (d) The probability the petition is not eventually approved is the probability it is not approved by each of three committees, which is $(1-p)^3 = 27/64$. Thus, a petition is eventually approved with probability $37/64$.

3. (a)

$$\begin{aligned} P(\text{exactly one}|\text{at least one}) &= \frac{P\{(\text{exactly one}) \cap (\text{at least one})\}}{P\{\text{at least one}\}} \\ &= \frac{P\{\text{exactly one}\}}{P\{\text{at least one}\}} \\ &= \frac{\binom{4}{1}(0.5)(0.5)^3}{1-(0.5)^4} = \frac{\frac{4}{16}}{\frac{15}{16}} = \frac{4}{15} \end{aligned}$$

   (b) There are 18 flights per day, so 5/18 are American, 4/18 are AirTrans, and 9/18 are Delta. These fractions give the pmf for the airline associated with a random flight. Thus, by the law of total probability, the probability a randomly selected flight departs late is

$$P\{\text{flight departs late}\} = \frac{5}{18} \cdot 0.2 + \frac{4}{18} \cdot 0.05 + \frac{9}{18} \cdot 0.1 = \frac{2.1}{18} = \frac{7}{60}.$$

   (c)

$$\begin{aligned} P(\text{flight is American}|\text{flight departs late}) &= \frac{P\{\text{flight is American and departs late}\}}{P\{\text{flight departs late}\}} \\ &= \frac{\frac{5}{18} \cdot 0.2}{\frac{7}{60}} = \frac{\frac{1}{18}}{\frac{7}{60}} = \frac{10}{21}. \end{aligned}$$

4. (a) The likelihood that $X = 4$ is $\frac{\lambda^4 e^{-\lambda}}{4!}$, and $\widehat{\lambda}_{ML}(4)$ is the value of λ that maximizes the likelihood, or equivalently, maximizes $\lambda^4 e^{-\lambda}$. Since $(\lambda^4 e^{-\lambda})' = (4\lambda^3 - \lambda^4)e^{-\lambda} = (4-\lambda)\lambda^3 e^{-\lambda}$, the likelihood is increasing in λ for $\lambda < 4$ and decreasing in λ for $\lambda > 4$; it is maximized at $\lambda = 4$. Thus, $\widehat{\lambda}_{ML}(4) = 4$.

   (b) This is the same as the probability exactly two even numbers show. Each die shows an even number with probability 0.5, so the number of even numbers showing, X, has the binomial distribution with parameter $p = 0.5$. Therefore, $P\{X = 2\} = \binom{4}{2}(0.5)^2(0.5)^2 = \frac{6}{16} = \frac{3}{8}$.

   (c) Each roll does not produce two even and two odd numbers with probability 5/8, and for that to happen on three rolls has probability $(5/8)^3 = \frac{125}{512}$.