University of Illinois Fall 2010

ECE 313: Hour Exam I

Monday October 11, 2010
7:00 p.m. — 8:00 p.m.
Room 141 Wohlers Hall & Room 114 David Kinley Hall

1. [**25 points**] A certain town has three newspapers: A, B, and C. The probability distribution for which newspapers a person in the town reads is shown in the following Karnaugh map:

B^c		B		
0.68	0	0.03	0.19	A^c
0.01	0.01	0.01	0.07	A
C^c	C	C	C^c	

   (a) [**5 points**] Find the probability a person reads only newspaper A.
   (b) [**5 points**] What is the probability a person reads at least two newspapers?
   (c) [**5 points**] What is the probability a person doesn't read any newspaper?
   (d) [**5 points**] If A and C are morning papers and B is an evening paper, what is the probability a person reads at least one morning paper plus an evening paper?
   (e) [**5 points**] What is the probability a person reads only one morning and one evening paper?

2. [**25 points**] A committee of three judges is randomly selected from among ten judges. Four of the ten judges are tough; the committee is tough if at least two of the judges on the committee are tough. A committee decides whether to approve petitions it receives. A tough committee approves 50% of petitions and a committee that is not tough approves 80% of petitions.

   (a) [**7 points**] Find the probability a committee is tough.
   (b) [**6 points**] Find the probability a petition is approved.
   (c) [**6 points**] Suppose a petition can be submitted many times until it is approved. If a petition is approved with probability 3/4 each time, what is the mean number of times it has to be submitted until it is approved?
   (d) [**6 points**] If instead, a petition can be submitted a maximum of three times, and the probability of approval each time is 1/4, find the probability a petition is eventually approved.

3. [**25 points**] The first part of this problem is not related to the second and third parts.

   (a) [**8 points**] Given that a family of four children has at least one girl, what is the probability it has exactly one girl? (Assume each child is a girl with probability 0.5, independently of the others.)
   (b) [**8 points**] Three airlines fly out of the Bloomington airport:
      - American has five flights per day; 20% depart late,

- AirTrans has four flights per day; 5% depart late,

- Delta has nine flights per day; 10% depart late.

What fraction of flights flying out of the Bloomington airport depart late?

(c) **[9 points]** Given that a randomly selected flight departs late (with all flights over a long period of time being equally likely to be selected) what is the probability the flight is an American flight?

4. **[25 points]** The first part of this problem is not related to the second and third parts.

(a) **[9 points]** Suppose a random variable X is assumed to have a Poisson pmf, but the parameter λ is unknown. Suppose it is observed that $X = 4$. What is the maximum likelihood (ML) estimate of λ, given this observation. Note: You should know the Poisson pmf – it will not be provided.

(b) **[8 points]** Suppose four dice are simultaneously rolled. What is the probability that two even and two odd numbers show?

(c) **[8 points]** Suppose four dice are repeatedly simultaneously rolled. What is the probability that strictly more than three rolls are needed until two even and two odd numbers show on the same roll? (A simultaneous roll of all four dice is counted as one roll.)