ECE 313: Problem Set 6
Bayes' Formula and Midterm Review Problems

Due:	Wednesday October 7 at 4 p.m.
Reading:	Powerpoint Lecture Slides, Sets 13-16
Noncredit Exercises:	**Chapter 3:** Problems 1, 2, 5, 10, 12, 16, 31, 38, 39, 51, 52 Theoretical Exercises 1, 2, 8, 16; Self-Test Problems 1-14.
Reminders:	No class on Wednesday September 30 and Friday October 2 but office hours as usual next week

1. **[Conditional Probabilities]**
This is Theoretical Exercise 3.5 in Ross.
We say that event B gives *positive information* about event A if $P(A \mid B) > P(A)$, that is, the occurrence of B makes the occurrence of A more likely.

   Now suppose that B gives positive information about A. If so,

   (a) Does A give positive information about B?

   (b) Does B^c give *negative information* about A, that is, is it true that $P(A \mid B^c) < P(A)$?

   (c) Does B^c give positive information or negative information about A^c?

2. **[Hindsight]**
The probability that a light bulb manufactured by Transylvania Corp. burns out during the n-th hour of operation is $p_1(n)$, $n = 1, 2, \ldots$. The probability that a light bulb manufactured by Eastinghouse Corp. burns out during the n-th hour of operation is $p_2(n)$, $n = 1, 2, \ldots$. Note that we are not giving you numerical values of these probabilities but do not forget that $\sum p_1(n) = \sum p_2(n) = 1$.

   A bulb is equally likely to have been made by one of the two manufacturers.

   Express your answers to the following questions in terms of $p_1(n)$ and $p_2(n)$.

   (a) What is the probability that the bulb burns out during the M-th hour of operation?

   (b) Given that the bulb burned out during the M-th hour of operation, what is the probability that it was manufactured by Transylvania Corp.?

   (c) Given that the bulb is still burning at the end of the M-th hour of operation, what is the probability that it was manufactured by Transylvania Corp.?

3. **[A woman is twice as good as a man!]**
Alice and Bob play the following game. First, Alice rolls a fair die and then Bob rolls the fair die. If Bob rolls a number at least as large as Alice's number, he wins the game. But if Bob rolled a number smaller than Alice's number, then Alice rolls the die again. If her second roll gives her a number that is less than or equal to Bob's number, the game ends with no winner (a tie, or draw as the British call it). If her second roll gives a number larger than Bob's number, Alice wins the game.

   Find the probability that Alice wins the game and the probability that Bob wins the game. Also, find the probability of a tie directly (and not as $P(\text{tie}) = 1 - P(\text{Alice wins}) - P(\text{Bob wins})$.) If the three probabilities do not add up to 1, explain.

4. **[Events and K-maps]**
Consider 7 independent flips of a coin and the events

$$A = \{\text{3rd flip is a head}\}$$
$$B = \{\text{5th flip is a head}\}$$
$$C = \{\text{5th and 7th flip are heads}\}$$
$$D = \{\text{5th or 7th flip are heads}\}.$$

(a) Find all pairs of events that are mutually exclusive.

(b) Find all pairs of events that are independent.

(c) Find all pairs of events such that one event contains the other. Be sure to state which event is a subset of the other.

5. **[The noisy cereal problem]**
The manufacturer of a cereal tests samples from the production line to see if the samples snap, crackle, and pop as advertised. Let A, B, and C respectively denote the events that the sample being tested *does not* snap, *does not* crackle, and *does not* pop. The manufacturer's tests show that $P(A) = P(C) = 0.3$, $P(B) = 0.2$, $P(AB \cup AC \cup BC) = 0.3$, $P(A \cap B \cap C) = 0.05$, $P(A \cap B) = 0.1$, and $P(A \cap C) = 2P(B \cap C)$.

(a) Sketch the sample space and indicate on it the events A, B, and C.

(b) What is the probability that the cereal snaps, crackles, and pops?

(c) Cereal that fails exactly one test is sold to discount supermarket chains to be marketed under the names Soggies, Blecchies, and Mushies. What is the probability that the sample fails *only* the snap test? *only* the crackle test? *only* the pop test?

6. **[Optimizing Flight Durations]**
Since there is no direct flight from San Diego (S) to New York (N), every time Alice wants to go from San Diego to New York, she has to connect through either Chicago (C) or Denver (D). There are flights every hour on the hour on the SC, SD, CN, and DN routes. Due to bad weather conditions, the SC flight can have a delay of one hour with probability p (and is on time with probability $1-p$). If Alice's SC flight is late, she gets on the *next* CN flight (i.e., leaves an hour later than scheduled from Chicago) but the CN flight she takes also can have a delay of one hour with probability p, and independent of the delay or on-schedule arrival of the SC flight. A similar situation holds at Denver except that both incoming and outgoing flights are independently subject to *two* hour delays with probability q (and are on time with probability $1-q$), and thus if Alice's SD flight is late, she leaves Denver two hours behind schedule. Note that Alice has SuperPlatinum status on Unirican Airlines and is guaranteed to get on the next flight to New York even if someone else has to be bumped to make room for her.

Note: DO NOT assume that $q = 1 - p$ as in the notation often used in this course.

(a) Find the probabilities that Alice arrives in N on schedule, an hour behind schedule, and two hours behind schedule if she chooses the SCN route. How late is she on average if she flies SCN?

(b) Find the probabilities that Alice arrives in N on schedule, two hours behind schedule, and four hours behind schedule if she chooses the SDN route. How late is she on average if she flies SDN?

Now assume that Alice chooses the SCN and SDN routes with equal probability when making a trip to New York.

(c) On average, how much behind schedule is Alice when she arrives in New York?

(d) Suppose Alice arrives in New York two hours behind schedule. What is the probability that she flew the SCN route?

(e) Suppose that Alice wants to maximize the probability that she arrives in New York less than 2 hours behind schedule. Under what conditions on p and q is the SCN route a better choice than the SDN route?

(f) Suppose now that Alice always flies the SCN route. On average, how many trips does she make before experiencing a 2 hour delay?