

ECE 313: Problem Set 8

Due: Wednesday, October 29, at 4 p.m.

Reading: Ross, Chapters 4 and 5.

Non-credit exercises: Ross, Chapter 5, Problems 24,25,30, and 35.

This Problem Set contains five problems

1. Which of the following functions $F(u)$ are valid CDFs? For those that are valid CDFs, compute the probability that the absolute value of the random variable exceeds 0.5.

$$\begin{aligned} \text{(a)} \quad F(u) &= \begin{cases} 0 & u < 0, \\ u^2, & 0 \leq u < 1, \\ 1, & u \geq 1. \end{cases} & \text{(b)} \quad F(u) &= \begin{cases} 0 & u < 1, \\ 2u - u^2, & 1 \leq u \leq 2, \\ 1, & u > 2. \end{cases} \\ \text{(c)} \quad F(u) &= \begin{cases} \frac{1}{2} \exp(2u) & u \leq 0, \\ 1 - \frac{1}{4} \exp(-3u), & u > 0, \end{cases} & \text{(d)} \quad F(u) &= \begin{cases} \frac{1}{2} \exp(2u) & u < 0, \\ 1 - \frac{1}{4} \exp(-3u), & u \geq 0, \end{cases} \end{aligned}$$

2. The number of hours that a student spends on ECE 440 homework is a random variable $\mathcal{X}$ with CDF

$$F_{\mathcal{X}}(u) = \begin{cases} 0, & u < 0, \\ (1+u)/8, & 0 \leq u < 1, \\ 1/2, & 1 \leq u < 2, \\ (4+u)/8, & 2 \leq u < 4, \\ 1, & u \geq 4. \end{cases}$$

Note that this is a *mixed* random variable: it takes on some values with nonzero probability (like a discrete random variable) but also takes on all values in intervals of the real line (like a continuous random variable).

- (a) Find $P\{\mathcal{X} = 2\}$, $P\{\mathcal{X} < 2\}$, $P\{\mathcal{X} > 2\}$, $P\{1 \leq \mathcal{X} \leq 3\}$, and $P\{\mathcal{X} > 2 \mid \mathcal{X} > 0\}$.
 (b) Find $E[\mathcal{X}]$.
3. Nine functions $f(u)$ are shown below. Note that in each case, $f(u) = 0$ for all u not in the interval specified. In each case,
- determine whether $f(u)$ is a valid probability density function (pdf).
 - If $f(u)$ is not a valid pdf, determine if there exists a constant C such that $C \cdot f(u)$ is a valid pdf.
- (a) $f(u) = 2u$, $0 < u < 1$. (b) $f(u) = |u|$, $|u| < \frac{1}{2}$
 (c) $f(u) = 1 - |u|$, $|u| < 1$, (d) $f(u) = \ln u$, $0 < u < 1$. *Hint:* $\ln u$ can be integrated by parts.
 (e) $f(u) = \ln u$, $0 < u < 2$, (f) $f(u) = \frac{2}{3}(u-1)$, $0 < u < 3$,
 (g) $f(u) = \exp(-2u)$, $u > 0$. (h) $f(u) = 4\exp(-2u) - \exp(-u)$, $u > 0$,
 (i) $f(u) = \exp(-|u|)$, $|u| < 1$,
4. Problem 11 on page 248 of Ross.
5. Problem 12 on page 248 of Ross.