University of Illinois Fall 2008

ECE 313: Hour Exam I

Monday October 13, 2008
7:00 p.m. — 8:00 p.m.
Room 1404 Siebel Center

1. [40 points] A store in the Paynless Shoe Sores chain has 13 different pairs of shoes in a barrel at the door. Suppose that **two** shoes are **picked at random** from the barrel. Note that this is sampling *without* replacement.

   (a) [18 points] What is the probability of getting a **matching pair** of shoes? What is the probability of getting **two left** shoes? What is the probability of getting a **left** shoe and a **right** shoe but **not** a **matching** pair of shoes?

   (b) [6 points] **Two more** shoes are picked at random from the barrel without replacing the first two shoes picked back in the barrel. What is the probability that these two shoes are a **matching pair** of shoes?

   (c) [8 points] What is the probability that there is **at least one matching pair** of shoes in the four that have been picked?

   (d) [8 points] Given that there are **no matching pairs** of shoes in the **four** that have been picked, what is the (conditional) probability that **all** four shoes are **left** shoes?

2. [20 points] Dilbert has **three** coins in his pocket, **two** of which are **fair**, and **one** of which is **biased** with $P(\text{Heads}) = \frac{3}{4}$.

   (a) [5 points] If Dilbert picks **two** coins out of his pocket what is the probability that he did **not** pick the **biased** coin?

   (b) [15 points] If Dilbert picks **two** coins out of his pocket, tosses **each** one **once**, and observes a Head and a Tail, what is the (conditional) probability that he did **not** pick the **biased** coin?

3. [25 points] Each box of Cornies, the breakfast of silver medalists, contains a picture of either Britney Spears or Paris Hilton. The purchase of each box of Cornies can be regarded as an independent trial of an experiment on which events S and H occur with probabilities $\frac{1}{4}$ and $\frac{3}{4}$ respectively.

   (a) [20 points] Let $\mathcal{X}$ denote the number of boxes of Cornies purchased till the experimenter has acquired **at least one picture** of each woman.
   What is $P\{\mathcal{X} = k\}$ for $k \geq 2$? What is $\mathsf{E}[\mathcal{X}]$?

   (b) [5 points] Let $\mathcal{Y}$ denote the number of boxes of Cornies purchased till the experimenter has acquired **at least two pictures** of each woman. What is $P\{\mathcal{Y} = 4\}$?

4. [15 points]

   (a) [9 points] If $\mathcal{X}$ is a **binomial** random variable with parameters $(4, \frac{1}{3})$, what are the **mean** and **variance** of the random variable $2 + 3\mathcal{X}$?

   (b) [6 points] Let $\mathcal{Y}$ be a **geometric** random variable with parameter p where the value of p is unknown. It is observed that $\{\mathcal{Y} = k\}$. What is the **maximum-likelihood estimate** $\hat{p}_{\text{ML}}$ of the parameter p?