

59. (a) $P(\text{40-year-old male survives to age 50}) = \exp \left[- \int_{40}^{50} \lambda(t) dt \right] = e^{-0.35} = 0.7047$
 (b) $P(\text{40-year-old male survives to age 60}) = \exp \left[- \int_{40}^{60} \lambda(t) dt \right] = e^{-1.21} = 0.2982$
60. (a) $P(\text{the item survives to age 2}) = 1 - F(2) = \exp \left[- \int_0^2 t^3 dt \right] = e^{-4} = 0.0183$
 (b) $P(0.4 \leq \text{lifetime} \leq 1.4) = \exp \left[- \int_0^{1.4} t^3 dt \right] - \exp \left[- \int_0^{0.4} t^3 dt \right] = 0.6109$
 (c) $P(\text{1-year-old item survives to age 2}) = \exp \left[- \int_1^2 t^3 dt \right] = e^{-3.75} = 0.0235$
- 61.

$$\begin{aligned} \frac{1}{\lambda(x)} &= \frac{1 - F(x)}{f(x)} = \frac{\int_x^\infty \lambda e^{\lambda t} (\lambda t)^{\alpha-1} dt}{\lambda e^{\lambda x} (\lambda x)^{\alpha-1}} \\ &= \int_x^\infty e^{\lambda(t-x)} \left(\frac{t}{x}\right)^{\alpha-1} dt = \int_0^\infty e^{\lambda z} \left(1 + \frac{z}{x}\right)^{\alpha-1} dz, \end{aligned}$$

where the last equality follows by a change of variables $z = t - x$. Note that the factors $\frac{1}{\Gamma(\alpha)}$ have been omitted since they cancel out. We see that the inverse of the hazard rate function is clearly decreasing in x when $\alpha \geq 1$ and decreasing when $\alpha \leq 1$. Hence, the desired result follows.

62. Note that we need to make at least 2 and at most 4 tests in total and at most 3 tests to identify the first defective transistor, so $2 \leq N_1 + N_2 \leq 4$ and $N_1 \leq 3$. In particular,

$$P_{(N_1, N_2)}(1, 1) = P_{(N_1, N_2)}(1, 2) = P_{(N_1, N_2)}(2, 1) = P_{(N_1, N_2)}(3, 0) = \frac{1}{10}$$

and

$$P_{(N_1, N_2)}(1, 3) = P_{(N_1, N_2)}(2, 2) = P_{(N_1, N_2)}(3, 1) = \frac{2}{10}.$$

63. (a) Yes, X and Y are independent.
 In particular, $f_X(x) = xe^{-x}$, $x > 0$ and $f_Y(y) = e^{-y}$, $y > 0$ and $f_{X,Y}(x, y) = f(x)f(y)$.
- (b) No, X and Y are not independent. In particular,

$$\begin{aligned} f_X(x) &= \int_x^1 f_{X,Y}(x, y) dy = 2(1 - x), \quad 0 < x < 1 \\ f_Y(y) &= \int_0^y f_{X,Y}(x, y) dx = 2y, \quad 0 < y < 1 \end{aligned}$$

and $f_{X,Y}(x, y) \neq f(x)f(y)$.

64. (a) $F_{A,B,C}(a, b, c) = abc$, $0 < a, b, c < 1$
 (b) The roots will be real if $B^2 \geq 4AC$. Now,

$$P(AC < x) = \int \int_{c \leq x/a, 0 \leq a \leq 1, 0 \leq c \leq 1} dadc = \int_0^x \int_0^1 dcda + \int_0^{x/a} \int_x^1 dx = x - x \log x$$

Hence, $F_{AC}(x) = x - x \log x$ and $f_{AC}(x) = -\log x$, $0 < x < 1$.

$$\begin{aligned} P(AC < B^2/4) &= \int_0^1 P(AC < b^2/4) f_B(b) db = \int_0^1 \int_0^{b^2/4} -\log x dx db \\ &= \int_0^1 \left[\frac{b^2}{4} - \frac{b^2}{4} \log \frac{b^2}{4} \right] db \\ &= \frac{\log 2}{6} + \frac{5}{36} \end{aligned}$$

where the above uses the identity

$$\int x^2 \log x dx = \frac{x^3 \log x}{3} - \frac{x^3}{9}.$$

65. Let X_i denote the number of type i events. Note $X = X_1 + X_2 + \dots + X_n$ and given X the X_i are multinomial (See Ex. 1f, p.247, Ross).

$$\begin{aligned} P(X_1 = r_1, \dots, X_n = r_n) &= P(X_1 = r_1 + 1, \dots, X_n = r_n, X = \sum_{i=1}^n r_i) \\ &= P(X_1 = r_1, \dots, X_n = r_n | X = \sum_{i=1}^n r_i) P(X = \sum_{i=1}^n r_i) \\ &= \frac{(\sum_{i=1}^n r_i)!}{r_1! r_2! \dots r_n!} p_1^{r_1} \cdot p_2^{r_2} \dots p_n^{r_n} \frac{e^{-\lambda} \lambda^{\sum_{i=1}^n r_i}}{(\sum_{i=1}^n r_i)!} \\ &= \sum_{i=1}^n \frac{e^{-\lambda p_i} (-\lambda p_i)^{r_i}}{r_i!} \end{aligned}$$

Hence, X_i are independent Poisson random variables with respective parameters λp_i .

66. (a) $P(W \leq t) = 1 - P(W > t) = 1 - P(X > t, Y > t) = 1 - [1 - F_X(t)][1 - F_Y(t)]$.
 (b) $f_W(t) = \frac{d}{dt} F_W(t) = f_X(t)[1 - F_Y(t)] + f_Y(t)[1 - F_X(t)]$
 Dividing by $[1 - F_X(t)][1 - F_Y(t)]$ now yields

$$\lambda_W(t) = \frac{f_X(t)}{[1 - F_X(t)]} + \frac{f_Y(t)}{[1 - F_Y(t)]} = \lambda_X(t) + \lambda_Y(t).$$