

1. (a) $P[A \cap B] = 0$ since $A \cap B = \emptyset$
 (b) $P[B] = P[A \cup B] - P[A] + P[A \cap B] = 1/4$
 (c) $P[A \cap B^C] = P[A] = 3/8$ since $A \subset B^C$.
 (d) $P[A \cup B^C] = P[B^C] = 3/4$ since $A \subset B^C$.
 (e) No, $P[A \cap B] = 0$ while $P[A]P[B] = 3/32$.
 (f) $P[D] = 2/3$ since $P[C \cap D] = P[C]P[D]$.
 (g) $P[C \cup D] = P[C] + P[D] - P[C \cap D] = 5/6$
 (h) $P[C \cap D^C] = P[C \setminus D] = P[C] - P[C \cap D] = 1/6$
 (i) $P[C \cup D^C] = 1 - P[D \setminus C] = 1 - P[D] + P[D \cap C] = 2/3$
 (j) $P[C^C \cap D^C] = P[(C \cup D)^C] = 1/6$
 (k) Yes, $P[C \cap D^C] = P[C]P[D^C] = 1/6$
2. $S = \{(n, x_1, \dots, x_{n-1}), n \geq 1, x_i \neq 6, i = 1, \dots, n-1\}$, with the interpretation that the outcome is $(n, x_1, \dots, x_{n-1})$ if the first 6 appears on roll n , and x_i appears on roll i , $i = 1, \dots, n-1$.
 $(\cup_{n=1}^{\infty} E_n)^C = \emptyset$
3. $A = \{1, 0001, 0000001, \dots\}$
 $B = \{01, 00001, 00000001, \dots\}$
 $(A \cup B)^C = \{001, 000001, \dots\} = C$, the event that C wins.
4. (a) $\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{6^5}$
 (b) $\frac{6 \cdot \binom{5}{2} \cdot 5 \cdot 4 \cdot 3}{6^5}$
 (c) $\frac{\binom{6}{2} \cdot 4 \cdot \binom{5}{2} \cdot \binom{3}{2}}{6^5}$
 (d) $\frac{6 \cdot 5 \cdot 4 \cdot \binom{5}{3}}{6^5}$
 (e) $\frac{6 \cdot 5 \cdot \binom{5}{3}}{6^5}$
 (f) $\frac{6 \cdot 5 \cdot \binom{5}{4}}{6^5}$
 (g) $\frac{6}{6^5}$
5. Define $F_i = E_i \cap \bigcap_{j=1}^i E_j^C$, for $1 \leq i \leq n$.
 Then F_i are pairwise disjoint and $\bigcap_{i=1}^n E_i = \bigcap_{i=1}^n F_i$.
6. The number of partitions that has $n+1$ and a fixed set of i of the elements $1, 2, \dots, n$ as a subset is T_{n-i} . Hence, as there are $\binom{n}{i}$ such subsets.

$$T_{n+1} = \sum_{i=0}^n \binom{n}{i} T_{n-i} = 1 + \sum_{i=0}^{n-1} \binom{n}{i} T_{n-i} = 1 + \sum_{k=1}^n \binom{n}{k} T_k.$$

7. (a) Bonferonni's Inequality simply follows from

$$1 \geq P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

- (b) Proof by induction on n .
Base step for $n = 1$ done in part (a).
By Bonferonni's Inequality

$$P(E_1 \dots E_n) \geq P(E_1 \dots E_{n-1}) + P(E_n) - 1$$

By the inductive hypothesis

$$P(E_1 \dots E_n) \geq \sum_{i=1}^n P(E_i) - (n-1) \geq \sum_{i=1}^n P(E_i) - (n-1)$$