

1. See solutions Problem Set #1 Fall 2000, Problem 3.
2. See solutions Problem Set #1 Fall 2000, Problem 4.
3. See solutions Problem Set #1 Fall 2000, Problem 6.
- 4.(a) By the generalized principle of counting there are
 $26 \times 26 \times 10 \times 10 \times 10 \times 10 \times 10 = 67,600,000$
- (b) $26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6 = 19,656,000$
5. Each kitten can be identified by a code number i, j, k, l where each of i, j, k, l is any of the numbers from 1 to 7. The number i represents which wife is carrying the kitten, j then represents which of that wife's 7 sacks contain the kitten; k represents which of the 7 cats in sack j of wife i is the mother of the kitten; and l represents the number of the kitten of cat k in sack j of wife i . By the generalized principle there are thus $7 \times 7 \times 7 \times 7 = 2401$ kittens.
6. $7!/(3! \times 4!) = 35$. Each path is a linear arrangement of 4 r 's and 3 u 's (r for right and u for up). For instance the arrangement r, r, u, u, r, r, u specifies the path whose first 2 steps are to the right, next 2 steps are up, next 2 are to the right, and final step is up.
7. There are $4!/(2! \times 2!)$ paths from A to the circled point; and $3!/(2! \times 1!)$ paths from the circled point to B. Thus, by the basic principle, there are 18 different paths from A to B that go through the circled point.
9. There are $(n \text{ choose } j)$ different 0 – 1 vectors whose sum is j , since any such vector can be characterized by a selection of j of the n indices whose values are then set equal to 1. Hence, there are sum of j from k to n $(n \text{ choose } j)$ vectors that meet the criterion.
10. There are $(n+m \text{ choose } r)$ groups of size r . As there are $(n \text{ choose } i) \times (m \text{ choose } r-i)$ groups of size r that consists of i men and $r-i$ women, we see that $(n+m \text{ choose } r) = \text{sum of } i \text{ from } 0 \text{ to } r (n \text{ choose } i) \times (m \text{ choose } r-i)$.
11. $(2n \text{ choose } n) = \text{sum of } i \text{ from } 0 \text{ to } n (n \text{ choose } i) \times (n \text{ choose } n-i) = \text{sum } i \text{ from } 0 \text{ to } n (n \text{ choose } i)^2$
12. The number of subsets of size k that have i as their highest numbered member is equal to $(i-k \text{ choose } k-1)$, the number of ways of choosing $k-1$ of the numbers $1, \dots, i-1$. Summing over i yields the number of subsets of size k .
13. See the text book "Solutions to Self-test Problems".