

Assigned: Wednesday, December 1

Due: Wednesday, December 8

Reading: Lectures 36, 37, 38 from Lecture notes on class webpage. Chapter 6, Sections 6.3, 6.4, 6.5, and 6.7 and Chapter 7, Sections 7.1-7.3 of Ross.

Noncredit Exercises: (Do not turn these in) Self-test problems #7, 13, 14 at the end of Chapter 6 of Ross, and Self-test problems #3, 4, and 7 at the end of Chapter 7.

Problems:

78. The jointly continuous random variables $\mathbf{X}$ and $\mathbf{Y}$ have joint pdf

$$f_{\mathbf{X},\mathbf{Y}}(u,v) = \begin{cases} 1/2, 0 \leq u < 1, 0 \leq v < 1, \text{ and } 0 \leq u+v < 1 \\ 3/2, 0 \leq u < 1, 0 \leq v < 1, \text{ and } 1 \leq u+v < 2 \\ 0, \text{ otherwise.} \end{cases}$$

Find $f_{\mathbf{X}}(u)$, $P\{\mathbf{X} + \mathbf{Y} \leq 3/2\}$ and $P\{\mathbf{X}^2 + \mathbf{Y}^2 \geq 1\}$.

79. Two resistors are connected in series to a one-volt voltage source as shown in the right-hand diagram above. Suppose that the resistance values $\mathbf{R}_1$ and $\mathbf{R}_2$ (measured in ohms) are independent random variables, each uniformly distributed on the interval $(0, 1)$. Find the pdf $f_{\mathbf{I}}(a)$ of the current $\mathbf{I}$ (measured in amperes) in the circuit.

80. Let $(\mathbf{X}, \mathbf{Y})$ have joint pdf $f_{\mathbf{X},\mathbf{Y}}(u, v) = \begin{cases} 2u, 0 < u < 1, 0 < v < 1, \\ 0, \text{ elsewhere.} \end{cases}$

Find the pdf of $\mathbf{Z} = \mathbf{X}^2\mathbf{Y}$.

81. The number of hours $\mathbf{R}$ that a student spends reading about probability in preparation for the ECE 413 Final Examination and the number of hours $\mathbf{S}$ that the student spends sleeping can be modeled as random variables with joint probability density function

$$f_{\mathbf{R},\mathbf{S}}(x,y) = \begin{cases} K, 10 \leq x+y \leq 20, x \geq 0, y \geq 0, \\ 0, \text{ otherwise.} \end{cases}$$

- (a) What is the value of K ?
- (b) What is the marginal pdf of $\mathbf{R}$?
- (c) Unfortunately, the more the student tries to read about probability, the more confused the student gets. Also, the less the student sleeps, the more tired the student gets. As a result, the student's *percentage* score $\mathbf{T}$ on the Final Exam is related to $\mathbf{S}$ and $\mathbf{R}$ via the equation

$$\mathbf{T} = 50 + 2.5(\mathbf{S} - \mathbf{R}).$$

Find the pdf of $\mathbf{T}$.

82. Let $E[\mathbf{X}] = 1$, $E[\mathbf{Y}] = 4$, $\text{var}(\mathbf{X}) = 4$, $\text{var}(\mathbf{Y}) = 9$, and $\rho_{\mathbf{X},\mathbf{Y}} = 0.1$

- (a) If $\mathbf{Z} = 2(\mathbf{X} + \mathbf{Y})(\mathbf{X} - \mathbf{Y})$, what is $E[\mathbf{Z}]$?
- (b) If $\mathbf{T} = 2\mathbf{X} + \mathbf{Y}$ and $\mathbf{U} = 2\mathbf{X} - \mathbf{Y}$, what is $\text{cov}(\mathbf{T}, \mathbf{U})$?
- (c) If $\mathbf{W} = 3\mathbf{X} + \mathbf{Y} + 2$, find $E[\mathbf{W}]$ and $\text{var}(\mathbf{W})$.
- (d) If $\mathbf{X}$ and $\mathbf{Y}$ are jointly Gaussian random variables, and $\mathbf{W}$ is as defined in (c), what is $P\{\mathbf{W} > 0\}$?

83. This problem has three independent parts. Do not apply the numbers from one part to the others.

- (a) If $\text{var}(\mathbf{X} + \mathbf{Y}) = 36$ and $\text{var}(\mathbf{X} - \mathbf{Y}) = 64$, what is $\text{cov}(\mathbf{X}, \mathbf{Y})$? If you are also told that $\text{var}(\mathbf{X}) = 3 \cdot \text{var}(\mathbf{Y})$, what is $\rho_{\mathbf{X},\mathbf{Y}}$?
- (b) If $\text{var}(\mathbf{X} + \mathbf{Y}) = \text{var}(\mathbf{X} - \mathbf{Y})$, are $\mathbf{X}$ and $\mathbf{Y}$ uncorrelated?
- (c) If $\text{var}(\mathbf{X}) = \text{var}(\mathbf{Y})$, are $\mathbf{X}$ and $\mathbf{Y}$ uncorrelated?