

Assigned: Wednesday, October 27

Due: Wednesday, November 3

Reading: Lectures 25, 26, 27 from Lecture notes on class webpage. Chapter 5, Sections 5.4 (pp. 199-210), 5.7 (pp. 223-225), and Chapter 9, Section 9.1 (pp. 432-435) of Ross.

Noncredit Exercises: (Do not turn these in) Self-test problems #9, 11 on pp. 236-237, #1, 2 on p. 454 of Ross.

Problems:

53. Consider a Poisson process with arrival rate μ . Let $\mathbf{X}$ denote the time of the first arrival after $t = 0$, and let τ denote a nonnegative real number.
- (a) What is $P\{\mathbf{X} > \tau\}$?
 - (b) What is the probability density function of $\mathbf{X}$?
 - (c) Let A denote the event that there are exactly four arrivals in the time interval $(0, 6]$. What is $P(A)$?
 - (d) Let B denote the event that there are exactly two arrivals in the interval $(4, 10]$. What is $P(AB)$?
 - (e) What is the conditional probability that $\{\mathbf{X} > \tau\}$ given that the event A occurred? Be sure to give the answer for all nonnegative values of τ .
- 54.(a) Problem 15, p. 229, Ross.
(b) Theoretical Exercise 9, p. 233, Ross.
55. Problem 16, p. 229, Ross.
56. Problem 30, pp. 230-231, Ross.
57. $\mathbf{X}$ is a continuous random variable with pdf $f_{\mathbf{X}}(u) = 0.5 \exp(-|u|)$, $-\infty < u < \infty$.
- (a) What is the value of $P\{\mathbf{X} \leq \ln 2\}$?
 - (b) Find the conditional probability that $P\{|\mathbf{X}| \leq \ln 2\}$ given that $\{\mathbf{X} \leq \ln 2\}$.
 - (c) Find the numerical value of $P\{\cos(\pi\mathbf{X}/2) < 0\}$.
 - (d) Now suppose that $\mathbf{X}$ denotes the voltage applied to a semiconductor diode, and that the current $\mathbf{Y}$ is given by $\mathbf{Y} = e^{\mathbf{X}} - 1$. Find the pdf of $\mathbf{Y}$.
58. The random variable $\mathbf{X}$ has probability density function $f_{\mathbf{X}}(u) = \begin{cases} 2(1-u), & 0 \leq u \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$
- Let $\mathbf{Y} = (1 - \mathbf{X})^2$.
- (a) What is the CDF $F_{\mathbf{Y}}(v)$ of the random variable $\mathbf{Y}$? Be sure to specify the value of $F_{\mathbf{Y}}(v)$ for all v , $-\infty < v < \infty$.
 - (b) What is the pdf $f_{\mathbf{Y}}(v)$ of the random variable $\mathbf{Y}$? Be sure to specify the value of $f_{\mathbf{Y}}(v)$ for all v , $-\infty < v < \infty$.