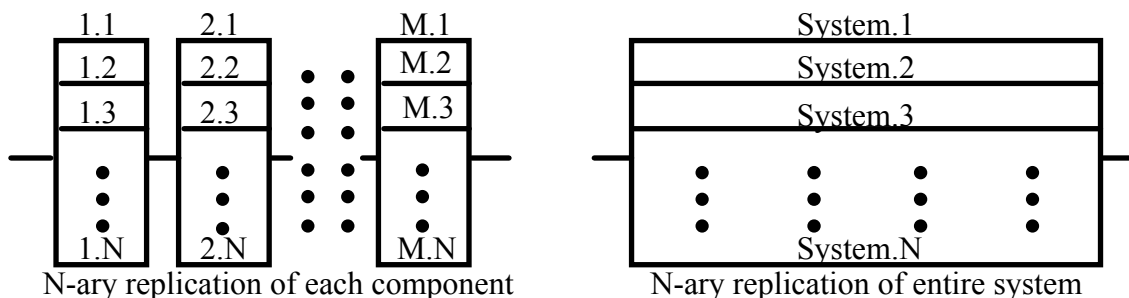


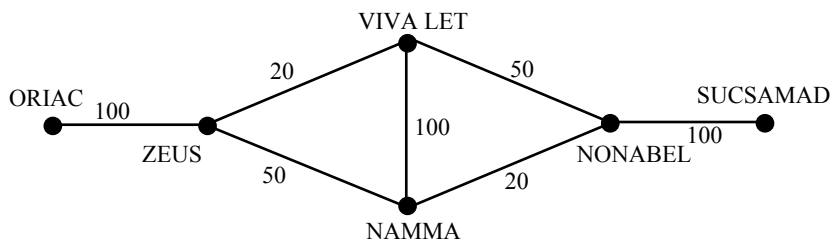
Assigned: Wednesday, October 13**Due:** Wednesday, October 20**Reading:** Lectures 19, 20, 21, 22 from Lecture notes on class webpage. Chapter 4, Section 4.9 (pp. 166-168), and Chapter 5, Section 5.1 (pp. 187-190) of Ross.**Noncredit Exercises:** (Do not turn these in) Self-test problems #1, 2 on p. 235 of Ross.**Problems:**

38. A system works if and only if all of its M components (numbered 1 through M) work. Each component fails (independently) with probability p . Consider two possible means of obtaining a more reliable system. We can replicate each component N times as shown in the graph model on the left. Or, we can replicate the entire system N times as shown in the graph model on the right. In either case, the result is called a replicated system. Note that both methods use the *same* number N of each component.



As a more concrete example of the question, consider which of the following two methods provides more reliable transportation

- a single gigantic car with N engines, N transmissions, N brakes, ... etc. that works (i.e. provides us with transportation) as long as at least one of its engines and at least one of its transmissions, and at least one of its brakes ... works.
 - N separate ordinary cars that fail as soon as any one of their components fail, but which together provide us with transportation as long as at least one car is in working condition.
- (a) For each model, find the probability of replicated system failure in terms of p , N and M .
- (b) Suppose that $M = 5$ and $p = 0.2$. If it is desired that the replicated system failure probability be less than 0.001, what should N be in each case?
- (c) Repeat part (b) assuming that $M = 1000$.
39. MiddleEast Bell, a division of Psingular Corp., has built a telephone network as shown below. Terrorists attack each of the seven links. The attacks may be considered to be independent events, and the attack on a link succeeds in severing the link with probability p . If a link is severed, switches automatically re-route calls so as to avoid the failed link (if possible).
- (a) What is the probability of being able to call from ORIAC to SUCSAMAD?
- (b) Given that it is possible to call from ORIAC to SUCSAMAD, what is the conditional probability that the ZEUS to NAMMA link is in working condition?
- (c) The link capacities (i.e., the numbers of telephone calls that the links can carry (in either direction)) are as marked on the diagram. Let X denote the number of calls that can be made from ORIAC to SUCSAMAD. Find the pmf and the expected value of X .



40. Which of the following are valid cumulative probability distribution functions (CDFs)? For those that are not valid CDFs, state at least one property of CDFs which is not satisfied. For those which are valid CDFs, compute $P\{|X| > 0.5\}$.

(a) $F_X(u) = \begin{cases} 0, & u < 1, \\ 2u - u^2, & 1 \leq u \leq 2, \\ 1, & u > 2. \end{cases}$ (b) $F_X(u) = \begin{cases} (1/2)\exp(2u), & u < 0 \\ 1 - (1/4)\exp(-3u), & u \geq 0 \end{cases}$

(c) $F_X(u) = \begin{cases} (1/2)\exp(2u), & u \leq 0, \\ 1 - (1/4)\exp(-3u), & u > 0. \end{cases}$

41. Let X denote the number of hours that a student works on ECE 413 each week. It is known that X is a *mixed* random variable with cumulative probability distribution function (CDF) $F_X(u)$ given by

$$F_X(u) = \begin{cases} 0, & u < 0 \\ (1+u)/8, & 0 \leq u < 1 \\ 1/2, & 1 \leq u < 2 \\ 1/2 + u/8, & 2 \leq u < 4 \\ 1, & u \geq 4 \end{cases}$$

Find the probability that the student

- (a) works for exactly 2 hours, (b) works for more than 2 hours,
 (c) works for less than 2 hours, (d) works for exactly 3 hours,
 (e) works for more than 1/2 but less than 3 hours,
 (f) works for more than 2 hours given that the student works at all, i.e. find $P\{X > 2 | X > 0\}$.

42. Which of the following are valid probability density functions? Assume that the functions are zero outside the ranges specified. For those which are not valid pdfs, state at least one property of pdfs which is not satisfied. Also, state whether there exists a constant C such that $Cf(u)$ is a valid pdf even though $f(u)$ is not.

- (a) $f(u) = |u|$ for $|u| < 1$. (b) $f(u) = 1 - |u|$ for $|u| < 1$.
 (c) $f(u) = \ln u$ for $0 < u < 1$, (d) $f(u) = \ln u$ for $0 < u < 2$. Hint: $\ln u$ can be integrated by parts
 (e) $f(u) = 2u$ for $0 < u < 1$. (f) $f(u) = (2/3)(u - 1)$ for $0 < u < 3$.
 (g) $f(u) = \exp(-2u)$, $0 < u < \infty$, (h) $f(u) = 4 \exp(-2u) - \exp(-u)$, $0 < u < \infty$.

43. Problem 4, p. 228 of Ross.

44. Problem 5, p. 228 of Ross.