

Assigned: Wednesday, October 6
Due: Wednesday, October 13

Reading Assignment: This problem set covers the topics of “Independent Events” and “Decision-Making Under Uncertainty”. The first topic is treated in Chapter 3, Section 3.4 (pp. 83 – 96) of Ross. For the second topic supplementary notes are posted on the course web-site. You are encouraged to attempt the self-test problems #12, 14, 16, 17 and 18 on pp. 120 – 121 of Ross; their solutions can be found in Appendix B.

The problems you will turn in, and which will be graded are the following:

31. Problem 54, p. 110, Ross
32. Problem 72, p. 113, Ross
33. Theoretical Exercise 6, p. 116, Ross
34. Theoretical Exercise 8, p. 116, Ross
35. We are given three events A, B, C for which $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{5}$, $P(C) = \frac{1}{4}$ and $P(ABC) = \frac{1}{120}$.
 - (a) Are A, B , and C independent?
 - (b) Suppose that the probability of ABC is instead $P(ABC) = \frac{1}{30}$. Are A, B , and C independent?
 - (c) Now suppose that in (a), you are also told that $P(AB) = \frac{1}{30}$, $P(AC) = \frac{1}{24}$, and $P(BC) = \frac{1}{10}$. Are A, B , and C independent?
36. In a factory there are two production lines manufacturing a certain electrical component. The first line A produces a defective component with probability 0.05, whereas the second line B is slightly inferior and produces a defective component with probability 0.10. When the factory fulfils orders for its customers it always specifies whether the batch was produced at line A or B . Suppose that there is an unlabeled batch consisting of 100 components. An inspection reveals that there are 7 defective components. What are the likelihoods that this batch was produced at line A or B ? What is the maximum-likelihood decision? State the maximum-likelihood decision rule in terms of the batch size N and the number of defective elements D .
37. Suppose you have 10 coins. You know that while 8 of these coins are fair the remaining 2 are biased, and that for the biased coins the probability of getting a head is 0.7. However, you cannot tell them apart. So you randomly pick a coin, flip it 5 times and count the number of heads in order to decide whether the coin is biased or not.
 - (a) Construct the likelihood matrix, state the ML decision rule and find the probability of making a wrong decision.
 - (b) Construct the joint probability matrix, state the MAP decision rule and find the probability of making a wrong decision for the MAP rule.
 - (c) Which decision rule is better suited for this problem. Why?