

Assigned: Wednesday, August 25

Due: Wednesday, September 1 (This is a no credit homework do not turn it in)

Problems: First three problems are based entirely on material covered in the *prerequisites* to this course. You should have mastered this stuff already, but may need to review the material one more time before starting the course. The problems below are assigned to help your review, and also as a self-diagnosis aid. If you cannot solve *all* these problems correctly, you will have difficulty in comprehending the material in the latter half of this course. It is not in your best interest to discover *after* the drop date that you really don't understand calculus as well as you thought you did, and that consequently you are in some danger of failing this course.

Do not use Mathematica or Matlab or a calculator etc. to do these problems except when you are specifically asked to do so.

1. In **this** problem, all **angles** are expressed in **radians**.

(a) Use your calculator to evaluate $\sqrt{52} \cos(3^{-1} \arctan(18\sqrt{3}/35))$.

(b) In this part, you have to find the limit of $\frac{1}{[\sin x]^2} - \frac{1}{x^2}$ as x approaches 0. Use your calculator to evaluate this function for *small* values of x say, $x = 10^{-1}$, $x = 10^{-2}$, $x = 10^{-3}$, etc. Does the function seem to be approaching a limit, and if so, what do you think is the limit? Now, use what you have learned about limits in calculus to find

$\lim_{x \rightarrow 0} \frac{1}{[\sin x]^2} - \frac{1}{x^2}$ analytically. (Hint: the answer is not 0, or 1, or ∞)

(c) Find the maxima of $f(x) = x^{25}(1.0001)^{-x}$ for $x > 0$. (If you have a graphing calculator, try it on this problem; otherwise just use standard calculus methods)

2.(a) What is the value of $\int_{-2}^1 |x| dx$? the value of $\int_0^1 x(1-x)^{19} dx$?

(b) Prove or disprove: there exists a function $f(x)$ satisfying both of the following two conditions:

(i) $0 \leq f(x) \leq 10$ for all real numbers x in the range $a \leq x \leq b$,
and

(ii) $\int_a^b f(x) dx < 0$.

(Hint: Does either function of part (a) satisfy both conditions?)

(c) Let $\frac{d}{dx} f(x) = g(x)$ for $-\infty < x < \infty$. Which of the following statements are true for all x , $-\infty < x < \infty$? C denotes an arbitrary constant.

(i) $\frac{d}{dx} f(-x) = g(-x)$. (ii) $\frac{d}{dx} f(x^2) = 2x g(x^2)$. (iii) $\frac{d}{dx} \exp(f(x^2)) = \exp(f(x^2)) g(x^2)$.

(iv) $\int g(-x) dx = -f(-x) + C$. (v) $\int g(x^2) dx = f(x^2)/(2x) + C$. (vi) $\int \frac{g(x)}{f(x)} dx = \ln(f(x)) + C$

(d) Evaluate $\int_0^{\infty} x \cdot \exp(-x^2/2) dx$

3.(a) Integrate $f(x, y) = \begin{cases} 6, & 0 < y < x, 0 < x < 1, \\ 0, & \text{elsewhere,} \end{cases}$ over the region $\{(x, y) : y > x^2\}$.

(b) Compute the integral of $(x^2 + y^2)^{-2}$ over the region $\{(x, y) : x^2 + y^2 > 2\}$.

From Ross, Chapter 1:

- 4. Problem 1.
- 5. Problem 6.
- 6. Problem 21.
- 7. Problem 22.
- 8. Problem 26.

From Ross, Chapter 1, Theoretical Exercises:

- 9. Problem 5.
- 10. Problem 8.
- 11. Problem 9.
- 12. Problem 11.

From Ross, Chapter 1, Self-test Problems:

- 13. Problem 7.