

UNIVERSITY OF ILLINOIS, URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 313: Probability with Engineering Applications-Fall 2003

SECOND MIDTERM EXAM NOVEMBER 20, 2003

Name _____

Section: 10:00 AM 11:00 AM

Score _____

Problem	Pts.	Score
1	36	
2	4	
3	5	
4	5	
5	10	
6	15	
7	13	
8	12	
Total	100	

Please do not turn this page over until told to do so.

You may not use any books, calculators, or notes other than two handwritten two-sided sheets of 8.5" x 11" paper.

GOOD LUCK!

(36 Pts.)

1. Answer TRUE or FALSE to each of the following questions:

(a) If $X \sim U[0, 1]$, then $X^2 \sim U[0, 1]$. **T/F**

(b) If $X \sim N(0, 1)$, then $Y = 2X + 1 \sim N(1, 4)$. **T/F**

(c) Let X be a uniform RV with mean μ and variance σ^2 and Y be a Gaussian RV with the same mean and variance. Then $P(|X - \mu| > \sigma) = P(|Y - \mu| > \sigma)$. **T/F**

(d) Let X be an exponential RV with parameter λ . Then $P(X > 5|X > 2) = e^{-3\lambda}$. **T/F**

(e) Let $X \sim N(0, \sigma^2)$ and $Y = X^2$. Then $E(Y) = \sigma^2$. **T/F**

(f) Let $F_X(u)$ be the CDF of a discrete RV X . If $F'_X(a) = 0$, then $P(X = a) \equiv 0$. **T/F**

(g) Let (X, Y) be a discrete random vector taking on values $\{(u_i, v_j), i, j = 0, 1, \dots\}$. Assume that the joint pmf of X and Y is $p_{X,Y}(u_i, v_j)$ and the pmf of X is $p_X(u_i)$. Then $p_X(u_i) \geq p_{X,Y}(u_i, v_j)$ for all v_j . **T/F**

(h) Let (X, Y) be a discrete random vector taking on values $\{(u_i, v_j), i, j = 0, 1, \dots\}$. Assume that the joint pmf of X and Y is $p_{X,Y}(u_i, v_j)$ and the conditional pmf of X given that $Y = v_j$ is $p_{X|Y}(u|v_j)$. Then $p_{X|Y}(u|v_j) \geq p_{X,Y}(u, v_j)$ for all values of u and v_j . **T/F**

(i) Let (X, Y) be a continuous random vector defined over the entire 2D plane. Assume that the joint pdf of X and Y is $f_{X,Y}(u, v)$ and the pdf of X is $f_X(u)$. Then $f_X(u) \geq f_{X,Y}(u, v)$ for all values of v . **T/F**

(j) Let (X, Y) be a continuous random vector defined over the entire 2D plane. Assume that the joint pdf of X and Y is $f_{X,Y}(u, v)$ and the conditional pdf of X given that $Y = v_0$ with $f_Y(v_0) > 0$ is $f_{X|Y}(u|v_0)$. Then $f_{X|Y}(u|v_0) \geq f_{X,Y}(u, v_0)$ for all values of u and v_0 . **T/F**

(k) Let X be a random variable characterizing the lifetime of a system with hazard rate function $\lambda(t)$. Then, $P(X > t) = \int_0^t \lambda(\tau) d\tau$. **T/F**

(l) Let X and Y be two random variables related to each other by $Y = g(X)$ with $g(\cdot)$ being a **monotonic** function. Then $P(Y > a) = P(X > g^{-1}(a))$ is always true. **T/F**

(4 Pts.)

2. If $X \sim U[0, 1]$ and $Y = 2X + 1$, then $Y \sim U[a, b]$ with a and b given below:

- (a) $[a, b] = [0, 1]$
- (b) $[a, b] = [1, 2]$
- (c) $[a, b] = [-1, -2]$
- (d) $[a, b] = [1, 3]$
- (e) $[a, b] = [-1, -3]$
- (f) None of the above

(5 Pts.)

3. Let $Y = |X|$, then

- (a) $E(Y) \geq E(X)$
- (b) $E(Y) \leq E(X)$
- (c) $E(Y) = |E(X)|$
- (d) None of the above

(5 Pts.)

4. Let $X \sim U[a, b]$ has mean μ and variance σ^2 . Determine a and b .

(10 Pts.)

5. Let $X \sim U[0, 1]$ and Y is an exponential RV with parameter λ . Assume that $Y = g(X)$. Determine $g(\cdot)$.

(15 Pts.)

6. Let X_1, X_2, \dots, X_n be independent RVs with the same exponential distribution with parameter λ . Determine $P(\min\{X_1, X_2, \dots, X_n\} < a)$, for a constant $a > 0$.

(13 Pts.)

7. You are given $\Phi(u)$, the CDF of a standard Gaussian RV, only for $u \geq 0$. Let $X \sim N(1, 2^2)$. Determine the following probabilities using the given values of $\Phi(u)$.

(a) $P(|X| < 3)$

(b) $P(X^2 - 3X + 2 < 0)$

(12 Pts.)

8. The discrete random variables X and Y have joint pmf $p_{X,Y}(u, v)$ given by

$v \downarrow u \rightarrow$	0	1	3	5
-1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	0
2	$\frac{1}{6}$	$\frac{1}{12}$	0	$\frac{1}{12}$
4	0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$

- (a) Find the marginal pmf $p_X(u)$ and $p_Y(v)$.
(b) Calculate $P(1 < X + Y < 4)$.