

Properties of Expectation

Assigned reading: *Ross* Chapter 7, Sections 1-5 (not starred sections).

Noncredit Exercises: pp. 379-389: 1-8, 10-12, 26, 38-40, 45, 50-51.

1. Let X and Y be jointly uniformly distributed on the unit square (with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$).
 - (a) Are X and Y independent? Are they uncorrelated?
 - (b) Let $W = X + Y$ and $Z = X - Y$. Are W and Z independent? Are they uncorrelated?
2. Let X and Y be jointly uniformly distributed on the square with vertices $(2, 0)$, $(1, 1)$, $(0, 0)$, $(1, -1)$.
 - (a) Are X and Y independent? Are they uncorrelated?
 - (b) Let $W = X + Y$ and $Z = X - Y$. Are W and Z independent? Are they uncorrelated?
3. Let the random variables X and Y be such that $E[X] = 1$, $E[Y] = 4$, $Var(X) = 4$, $Var(Y) = 9$, and $\rho = 0.1$. Let $W = 3X + Y + 2$.
 - (a) Find $E[W]$ and $Var(W)$.
 - (b) If X and Y are jointly Gaussian random variables, what is $P\{W > 0\}$?
4. Let X and Y be jointly Gaussian zero-mean random variables, with variances σ_X^2 and σ_Y^2 (respectively) and with correlation coefficient ρ . Find the angle θ such that the random variables $Z = X \cos \theta + Y \sin \theta$ and $W = -X \sin \theta + Y \cos \theta$ are *independent* Gaussian random variables. Note that this coordinate transformation corresponds to a rotation of the axes by θ . Thus, in this problem you will show how to transform two dependent Gaussian random variables to independent Gaussian random variables by a simple rotation.
5. Let X and Y be jointly Gaussian zero-mean random variables, with variances σ_X^2 and σ_Y^2 (respectively) and with correlation coefficient ρ . Show that the marginal density f_X is the zero-mean Gaussian density with variance σ_X^2 . (Hint: First factor out the terms that do not depend on u . Then, complete the square in the exponent of the joint density function. Finally, integrate the joint density to obtain the marginal for X .)
6. Suppose that the value of Y is to be estimated in terms of the random variables X_1 and X_2 by the linear predictor $g(X_1, X_2) = a + bX_1 + cX_2$. Further, suppose that $E[X_1] = E[X_2] = Cov(X_1, X_2) = 0$. Determine the values for a , b and c that minimize $E[(Y - (a + bX_1 + cX_2))^2]$.
7. Problem 73, page 388 of *Ross*.