

Jointly Distributed Random Variables

Assigned reading: *Ross* Chapter 6, Sections 1 and 4.

Noncredit Exercises: pp. 290-295: 1-8, 10-12, 35-41.

1. The joint pmf of X and Y is

$$p_{X,Y}(i, j) = \begin{cases} 2^{-(i-1)}3^{-j} & : i = 1, 2, \dots; j = 1, 2, \dots \\ 0 & : \text{otherwise} \end{cases}$$

- (a) Find the marginals of X and Y . What kinds of random variables are these?
 - (b) Find the conditional pmf of X given that $Y = k$.
 - (c) Are X and Y independent? Can you arrive at this conclusion from your answer to part (b)?
2. The discrete random variables X and Y have the joint pmf $p_{X,Y}(u, v)$ given by the following table.

p_{ij}		u_j			
		0	1	3	5
	4	0	1/12	1/6	1/12
v_i	3	1/6	1/12	0	1/12
	-1	1/12	1/6	1/12	0

For example, $p_{X,Y}(0, 3) = 1/6$.

- (a) Find the marginal pmfs $p_X(u)$ and $p_Y(v)$ of X and Y .
 - (b) Are the random variables X and Y independent.
 - (c) Find $P\{X \leq Y\}$ and $P\{X + Y \leq 8\}$.
3. The jointly continuous random variables X and Y have joint pdf given by

$$f_{X,Y}(u, v) = \begin{cases} 2e^{-(u+v)} & : 0 < u < v < \infty \\ 0 & : \text{otherwise} \end{cases}$$

- (a) Sketch the $u - v$ plane and indicate on it the region over which $f_{X,Y}(u, v)$ is nonzero.
 - (b) Find the marginal pdfs of X and Y .
 - (c) Find $P\{Y > 3X\}$.
 - (d) For $a > 0$, find $P\{X + Y \leq a\}$.
4. The jointly continuous random variables X and Y have joint pdf given by

$$f_{X,Y}(u, v) = \begin{cases} 1/2 & : 0 \leq u < 1, 0 \leq v < 1, 0 \leq u + v < 1 \\ 3/2 & : 0 \leq u < 1, 0 \leq v < 1, 1 \leq u + v < 2 \\ 0 & : \text{otherwise} \end{cases}$$

- (a) Find the marginal pdf of X .
- (b) Find $P\{X + Y \leq 3/2\}$.
- (c) Find $P\{X^2 + Y^2 \geq 1\}$.

5. Problem 9, pg. 290 of *Ross*.

6. The number of α -particles emitted by a source during a unit time interval can be modeled as a Poisson random variable X with parameter λ . The α -particles are detected by means of a Geiger counter which detects a particle with probability $p < 1$ (i.e., the Geiger counter is not 100% accurate). The detections of the particles can be considered independent events. Thus, if n particles have been emitted, the Geiger counter reading can be modeled as a binomial random variable Y with parameters (n, p) . Thus, the conditional pmf of Y given that $X = n$ is a binomial pmf

$$p_{Y|X}(k|n) = \binom{n}{k} p^k (1-p)^{n-k}, \quad 0 \leq k \leq n.$$

Note that $Y \leq X$, i.e., the counter never mistakenly counts a particle that was not emitted.

- (a) Sketch the $u-v$ plane and the joint pmf of X and Y . Precision is not required in the sizes of the blobs that you draw to represent probability mass, but be sure that you do not indicate nonzero probability mass where there should be no probability mass.
- (b) What is the unconditional pmf of Y ?
- (c) What is the conditional pmf of X given $Y = k$?