

Events and Combinatorics

Assigned reading: *Ross* Chapter 1, Chapter 2 sections 1-5.

Noncredit Exercises: pp.53-58: 3-5, 8-14, 23, 27-29, 36, 38, 39, 41, 43, 45, pp.59-64: 4, 5, 7, 11, 18

1. Events and their combinations

Express each of the following events in terms of the events A , B , and C , and the operations of complementation, union, and intersection:

- (a) at least one of the events A, B, C occurs;
- (b) at most one of the events A, B, C occurs;
- (c) none of the events A, B, C occurs;
- (d) all three events A, B, C occur;
- (e) exactly one of the events A, B, C occurs;
- (f) events A and B occur, but not C ;
- (g) either event A occurs, or if not then B also does not occur.

In each case draw the corresponding Karnaugh maps.

2. Counting events I

Eight islanders are in competition for a large sum of money. Some of the islanders consider the possibility of forming alliances to improve their chances of winning.

- (a) Only one alliance is formed, and it contains at least two people. How many such alliances are there?
- (b) How many of these possible alliances contain Rich?
- (c) Two of the islanders, Kelly and Sue, don't really like one another, and refuse to join the same alliance. Once again, one alliance is formed, and it contains at least two people (but not both of Kelly and Sue). How many such alliances can be formed?
- (d) How many of these alliances that do not contain both Sue and Kelly do contain Rich?
- (e) If the islanders form two disjoint alliances of four people each, a deadlock will occur. How many ways can this occur?

3. Counting events II

Continuing with the above scenario, each of the eight islanders will cast one vote for either Kelly or Rich.

- (a) How many possible results are there for the voting? (Note: a result is defined as the pair (n_r, n_k) , where n_r is the number of votes for Rich and n_k is the number of votes for Kelly.)
- (b) Sue states that she will not vote for Kelly. Now how many possible results are there?
- (c) Rich wins if he receives at least five votes. How many assignments of votes to islanders produce a win for Rich?
- (d) How many assignments of votes to islanders produce a tie (i.e., four votes for Kelly and four votes for Rich)?

4. Orderings I

A large urn contains seven slips of paper. The name *Rich* is written on four of the slips, and the name *Kelly* is written on remaining three. The slips are drawn at random, one by one until the urn is empty.

- (a) In how many possible orders can the two names be drawn?
- (b) A dramatic finish occurs when *Rich* is the last name drawn. What is the probability of a dramatic finish?
- (c) What is the probability that *Rich* will be drawn on each of the first four draws?

5. Orderings and constraints

At the tribal council, seven islanders sit in a row. How many ways can the islanders be seated for each of the following conditions?

- (a) There are no constraints on seating order.
- (b) Rudy and Rich must be seated next to one another.
- (c) No two of Rudy, Sue, Kelly or Gervase may be seated next to one another.
- (d) Colleen and Greg must not be seated next to each other.
- (e) All of constraints b,c and d must be satisfied.

6. Events and their probabilities I

There are seven supermarkets in a town. Three visitors pick supermarkets at random to shop. Find the probability for each of the following events.

- (a) They all choose the same supermarket.
- (b) They choose three different supermarkets.
- (c) They do not all choose the same supermarket.
- (d) At least two of them choose the same supermarket.

7. Events and their probabilities II

An experiment consists of picking a student from the set of all students registered on the UIUC campus this semester. It is *not* necessary to assume that all students are equally likely to be picked, but you may make this assumption if you like.

- (a) Consider the two events:

A — the student has had four years of high school science (FYS),
 B — the student has had calculus in high school.

If the probability that the student has had at least one of FYS and calculus is 0.6, and the probability that the student has missed at least one of the two is 0.8, what is the probability that the student has had *exactly* one of the two?

- (b) Let C denote the event that the student is registered in ECE313 this semester, and let events A and B and their probabilities be as in part (a). If students who have had at most one of FYS and calculus did not register in ECE313 this semester,
 - i. What is $P(A^c \cap B^c \cap C^c)$?
 - ii. What is the probability that the student picked is not registered in ECE313 *and* has had exactly one of FYS or calculus in high school?
- (c) Using the data given in parts (a) and (b), which of the following probabilities:

$$P(ABC), P(C), P(A^c B C^c), P(A), P(A^c B^c C), P(A \cup B \cup C^c)$$

can you compute? It is not necessary to actually compute each probability.