Solutions to ECE313 Exam 2, Fall 2002

1. (a)

$$a = \int_1^4 \frac{1}{x} dx = \ln 4$$

(b)

$$E[X] = \int_1^4 \frac{x}{ax} dx = \frac{3}{a} = \frac{3}{\ln 4}$$

(c)

$$\int_3^5 f(x)dx = \int_3^4 \frac{1}{ax} dx = \frac{\ln 4 - \ln 3}{\ln 4}$$

2. (a) Differentiate answer to part (b) to get:

$$f_X(x) = \begin{cases} \frac{1}{2\sqrt{x}} & \text{if } 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

(Also need to sketch this function.)

(b) Note that X takes values in $[0, 1]$, so let $0 \leq x \leq 1$ and compute

$$P[X \leq x] = P[U^2 \leq x] = P[-\sqrt{x} \leq U \leq \sqrt{x}] = \sqrt{x}$$

so

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

(c) $E[X] = \int_{-1}^1 u^2 \frac{1}{2} du = \frac{1}{3}$

(d) $E[X^2] = \int_{-1}^1 u^4 \frac{1}{2} du = \frac{1}{5}$ so $Var(X) = \frac{1}{5} - (\frac{1}{3})^2 = \frac{4}{45}$

3. (a)

$$\Lambda(x) = \frac{f_1(x)}{f_0(x)} = \begin{cases} \frac{1}{5e^{-5x}} & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x \geq 1 \end{cases}$$

(b) Say H_1 is true if $f_1(X) > f_0(X)$, or $\Lambda(X) \geq 1$ or $\frac{\ln 5}{5} \leq X \leq 1$.

(c) $p_{false\_alarm} = P[\text{say } H_1 | H_0] = \int_{\frac{\ln 5}{5}}^1 5e^{-5x} dx = \frac{1}{5} - e^{-5}$

(d) $p_{miss} = P[\text{say } H_0 | H_1] = \int_0^{\frac{\ln 5}{5}} 1 dx = \frac{\ln 5}{5}$

4. (a) The number of girls, G, has the binomial distribution with parameters (400,0.5), and therefore mean 200 and variance 100. Thus

$$P[G \geq 205] = P[\frac{G - 200}{\sqrt{100}} \geq \frac{205 - 200}{\sqrt{100}}] \approx Q(0.5) = 0.3085$$

(b) The number of children X with birthday May 15 has the binomial distribution with parameters $(400, \frac{1}{365})$. Thus the distribution of X is approximately Poisson with mean $\lambda = \frac{400}{365} \approx 1.1$. Thus, the desired probability is $P[X = 2] \approx \frac{\lambda^2 e^{-\lambda}}{2} \approx \frac{1.2}{2}(.3329) \approx 0.2$

(c) The number of girls Y with birthday June 1 has the binomial distribution with parameters $(400, \frac{1}{2\times 365})$. Thus the distribution of Y is approximately Poisson with mean $\frac{400}{2\times 365} \approx .55$. Thus, the desired probability is $P[Y = 0] \approx e^{-.55} \approx 0.57$ (interpolating from the given table).