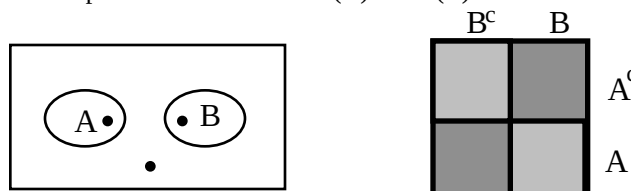


- 1.(a) It is easy to see from the left-hand diagram below that no matter what the outcome is, at least one of A and B did not occur. Hence, $P(\text{at least one of A and B did not occur}) = 1$. More formally,
 $P(\text{at least one of A and B did not occur}) = P(A^c \cup B^c) = P((A \cap B)^c) = P(\Omega - (A \cap B)) = P(\Omega) - P(A \cap B) = 1 - P(A \cap B)$.
 The answer does not at all depend on the values of $P(A)$ and $P(B)$.

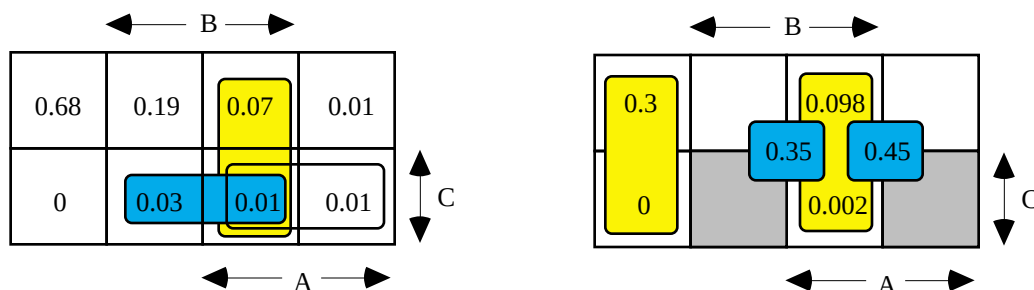


- (b) In the right-hand diagram above, $A \cup B$, the event that at least one of A and B occurred, has vertical stripes while $A^c \cup B^c$, the event that at least one of A and B did not occur, has horizontal stripes. The union of these is obviously Ω . The forever plaid region is $A \cap B = (A \cap B^c) \cup (A^c \cap B)$, the event that *exactly one* of A and B did not occur. Thus, we have that
 $1 = P(\Omega) = P((A^c \cup B^c) \cup (A \cap B)) = P(A^c \cup B^c) + P(A \cap B) - P((A^c \cup B^c) \cap (A \cap B))$
 $= P(A^c \cup B^c) + P(A \cap B) - P(A \cap B) = P(A^c \cup B^c) + P(A \cap B) - P(A \cap B)$.
 Hence, $P(A \cap B) = P(A^c \cup B^c) + P(A \cap B) - 1 = 0.4$.

2. This problem is best solved with a Venn diagram or more preferably a Karnaugh map as shown below. From the given values of $P(ABC) = 0.01$, $P(AB) = 0.08$, $P(BC) = 0.04$, and $P(AC) = 0.02$, it is easy to find $P(ABC^c) = 0.08 - 0.01 = 0.07$, $P(A^cBC) = 0.04 - 0.01 = 0.03$, and $P(AB^cC) = 0.02 - 0.01 = 0.01$. Next, since $P(A) = 0.1 = P(ABC) + P(ABC^c) + P(AB^cC) + P(AB^cC^c)$, we get $P(AB^cC^c) = 0.01$.

Similarly, we can show that $P(A^cBC^c) = 0.19$, and $P(A^cB^cC) = 0$. Finally, we can get
 $P(A \cap B \cap C) = P(ABC) + P(ABC^c) + P(A^cBC) + P(AB^cC) + P(AB^cC^c) + P(A^cBC^c) + P(A^cB^cC)$
 $= 0.01 + 0.07 + 0.03 + 0.01 + 0.01 + 0.19 + 0 = 0.32$. Crosscheck: Proposition 4.4 gives $P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$
 which is also 0.32. Finally, $P(A^cB^cC^c) = 1 - P(A \cap B \cap C) = 0.68$. The probabilities asked for are

- (a) $P(\text{only 1}) = P(AB^cC^c) + P(A^cBC^c) + P(A^cB^cC) = 0.01 + 0.19 + 0 = 0.2$,
 (b) $P(\text{at least 2}) = P(AB \cap BC \cap AC) = P(ABC) + P(ABC^c) + P(A^cBC) + P(AB^cC)$
 $= 0.01 + 0.07 + 0.03 + 0.01 = 0.12$. Crosscheck: $P(AB \cap BC \cap AC) = P(AB) + P(BC) + P(AC) - 2P(ABC)$
 (c) $P(B \text{ and at least one other}) = P(B \cap (A \cup C)) = P(AB) + P(BC) - P(ABC) = 0.08 + 0.04 - 0.01 = 0.11$,
 (d) $P(\text{no papers}) = P(A^cB^cC^c) = 1 - P(A \cap B \cap C) = 0.68$.

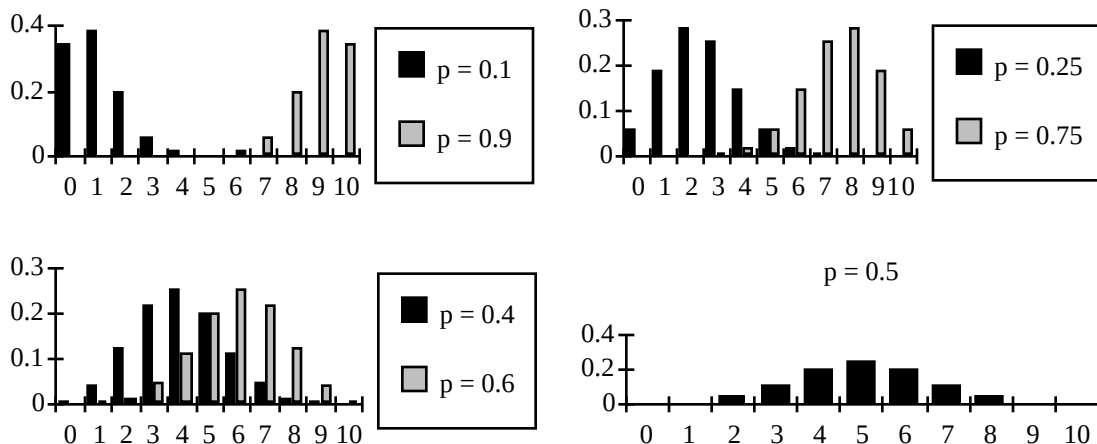


- 3.(a) From DeMorgan's theorem and $P(A^cB^c) = 0.3$, we get that $P(A \cap B) = 0.7$.
 $P(AB) = P(A) + P(B) - P(A \cap B) = 0.1$. $P(AB^c) = P(A) - P(AB) = 0.35$.
 (b) $P(ABC^c) = P(AB) - P(ABC) = 0.098$. Note that C can be partitioned into ABC , $(A \cap B)C$ and A^cB^cC , where $(A \cap B)C$ is the shaded set in the right-hand figure shown above. Since $P(A^cB^cC) = 0$, we get that
 $P(C) = P(ABC) + P((A \cap B)C) = 0.006$.
 (c) $P(A^cBC^c)$ and $P(A \cap C)$ cannot be computed.
 4. Since A is a subset of $A \cap B \cap C$, we know that $P(A) \leq P(A \cap B \cap C)$. Similar inequalities hold for B and C as well. Adding the three inequalities gives that $P(A) + P(B) + P(C) \leq 3P(A \cap B \cap C)$, and thus it follows that $[P(A) + P(B) + P(C)]/3 \leq P(A \cap B \cap C)$.
 To prove the other inequality, note that $P(E \cup F) = P(E) + P(F) - P(EF)$ for *any* events E and F, and hence $P(A \cap B) \leq P(A) + P(B)$ holds for events A and B (with equality if $P(A \cap B) = 0$.) Let D denote $(A \cap B)$. Then, $P(A \cap B \cap C) = P(D \cap C) \leq P(D) + P(C) = P(A \cap B) + P(C) \leq P(A) + P(B) + P(C)$.
 5. $P(A \cap (B^c \cap C^c)^c) = P(A \cap (B \cap C))$ by DeMorgan's theorem.

- (a) $P(B \cap C) = 0$, and therefore $P(A \setminus (B \cap C)) = P(A \setminus \emptyset) = P(A) = 1/3$.
 (b) $P(A \setminus (B \cap C)) = P(A) + P(B \cap C) - P(A \cap B \cap C) = 1/2 + 1/4 - 1/8 = 5/8$.
 (c) $P(A \setminus (B \cap C)) = P(A) + P(B \cap C) - P(A \cap B \cap C) = 1/2 + 1/3 - 0 = 5/6$. (Why?)
 (d) $(A^c \cap (B^c \setminus C^c))^c = A \setminus (B^c \setminus C^c)^c$ by DeMorgan's theorem. Hence,
 $P(A \setminus (B^c \setminus C^c)^c) = 1 - P(A^c \cap (B^c \setminus C^c)) = 1 - 0.6 = 0.4$.

- 6.(a) Each subset of two teams plays one game. Hence the number of games played is $\binom{5}{2} = 10$.
 (b) Yes. Arrange the 5 teams in a circle. Measuring distances along the circumference of the circle, each team can be thought of as having two teams “on its left” and the other two teams “on its right.” Furthermore, team A has team B “on its right” if and only if team B has team A “on its left.” Then, the rule “wear dark uniforms when playing a team on your left; wear light uniforms when playing a team on your right” satisfies the desired conditions.
 (c) Let A be the event that *some* team has a 4-0 record and B the event that *some* team has a 0-4 record. We are asked for the probability of the event $A^c \cap B^c$, that no team has a 4-0 record and no team has 0-4 record. But, by DeMorgan's laws, $A^c \cap B^c = (A \cup B)^c$, and hence $P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$. Now, let A_i be the event that the i -th team has a 4-0 record, and consider that *at most* one team can have a 4-0 record, that is, $A_i \cap A_j = \emptyset$ if $i \neq j$. Then, $A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$, and furthermore $P(A) = P(A_1) + P(A_2) + P(A_3) + P(A_4) + P(A_5)$. Similarly, let B_i be the event that the i -th team has a 0-4 record, etc. Now, for A_i to occur, the four games that the i -th team plays are wins for the i -th team, and it does not matter what happens in the other 6 games — all 2^6 possible results of those 6 games still give a 4-0 record for the i -th team. Since there are 2^{10} equally likely possible results of the 10 games, we get that $P(A_i) = 2^6/2^{10} = 1/16$, and thus $P(A) = 5/16$. Similarly, $P(B) = 5/16$ as well. Finally, $A \cap B = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \cap (B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5)$ is the union of 20 mutually exclusive terms of the form $A_i \cap B_j$ with $i \neq j$ (why only 20 and why mutually exclusive?). Furthermore, if $A_i \cap B_j$ occurs, we know the results of 7 of the 10 games (why not 8?) and don't care what happens in the other three. Thus, $P(A_i \cap B_j) = 2^3/2^{10} = 1/128$, $P(A \cap B) = 20/128 = 5/32$, and thus $P(A^c \cap B^c) = 1 - [5/16 + 5/16 - 5/32] = 17/32$, i.e. slightly more than 1/2.

7.(a), (b)



- (c) From the table, the probabilities for any given value of p and $1-p$ are just “reverses” of each other in the sense that $P\{X = k\}$ for probability p is just $P\{X = 10-k\}$ for probability $1-p$.
 (d) If X is a binomial random variable with parameters (n, p) , then it counts the number of times that an event A of probability p occurred on n trials. Let Y denote the random variable that counts the occurrences of A^c on these n trials. Then, obviously Y is a binomial random variable with parameters $(n, 1-p)$, and equally obviously $Y = n - X$, so that $P\{X = k\} = P\{Y = n-k\}$ exactly as observed in the table and part (c).
 (e) For $p = 0.1, 0.25, 0.4, 0.5, 0.6, 0.75$, and 0.9 , $P\{X = k\}$ is largest for $k = 1, 2, 4, 5, 6, 8$, and 9 respectively exactly as predicted by the theory (would Ross lie to you?)
- 8.(a) $P(\text{same on all three days}) = (0.2)^3 + (0.5)^3 + (0.3)^3 = 0.16$.
 (b) $P(\text{same on two of three days}) = 3 \times \{(0.2)^2 \times [1 - 0.2] + (0.3)^2 \times [1 - 0.3] + (0.5)^2 \times [1 - 0.5]\} = 0.66$.
 (c) $P(\text{different on all three days}) = 3! \times [0.2 \times 0.5 \times 0.3] = 0.18$. Alternatively, we can compute this as $1 - 0.16 - 0.66$. (Why?)