ECE 313 Fall 2001
SECOND HOUR EXAMINATION
Monday November 12, 2001, 11:00 a.m. — 11:50 a.m.

1. **(16 points)** Check **one box** for each of the following questions. No justification is required, but, your score will be reduced by 2 points for each wrong answer (You get +8 points for each right answer; 0 for no answer).

(a) Which of the following four statements are **NOT** properties of **all** CDFs?

‹ $P\{\mathbf{X} > b\} = 1 - F_{\mathbf{X}}(b)$. ` If $F_{\mathbf{X}}(a) < F_{\mathbf{X}}(b)$, then $a < b$.

´ If $a < b$, then $F_{\mathbf{X}}(a) < F_{\mathbf{X}}(b)$. ^ $F_{\mathbf{X}}(u) = 1/2$ for some u, $-\infty < u < \infty$.

- ℞ **Only** ` is not a property of CDFs.
- ℞ **Only** ` and ´ are not properties of CDFs.
- ℞ **Only** ´ is not a property of CDFs.
- ℞ **Only** ´ and ^ are not properties of CDFs.
- ℞ You blew it, Professor! **All four** are properties of CDFs.

(b) $f_{\mathbf{X}}(u)$ denotes the probability density function (pdf) of a continuous random variable $\mathbf{X}$. Which of the following properties are satisfied by all pdfs?

‹ $f_{\mathbf{X}}(u) \le 1$ for all u, $-\infty < u < \infty$. ` $\lim_{u \to -\infty} f_{\mathbf{X}}(u) = 0$

´ $\lim_{u \to \infty} f_{\mathbf{X}}(u) = 1$ ^ $P\{a < \mathbf{X} < b\} = P\{a \le \mathbf{X} \le b\}$

- ℞ **Only** ‹ , ` , and ^ are properties of all pdfs.
- ℞ **Only** ` and ^ are properties of all pdfs.
- ℞ **Only** ` , ´ and ^ are properties of all pdfs.
- ℞ **All four** are properties of all pdfs.
- ℞ None of the above. Only the following are properties of all pdfs:_____

2. **(12 points)** $\mathbf{X}$ denotes a Gaussian random variable with mean 2 and variance 25.

Find $P\{|\mathbf{X} - 4| > 3\}$ and $P\{\mathbf{X} < 3 \mid \mathbf{X} > 2\}$ using the table of values of the unit Gaussian CDF $\Phi(\bullet)$ on the last page of this exam booklet.

3. **(20 points)** Let $\mathbf{X}$ denote a random variable uniformly distributed on the interval [–1, 4], and let $\mathbf{Y} = |\mathbf{X}-1|$

(a) **(6 points)** Find E[$\mathbf{Y}$].

(b) **(14 points)** Find $f_{\mathbf{Y}}(v)$, the probability density function of $\mathbf{Y}$.

4. **(20 points)** Consider a Poisson process with arrival rate μ.

(a) **(5 points)** What is the probability that there are 2 arrivals in the time interval (3, 6] ? i.e., what is the probability that the random variable $\mathbf{N}(3, 6]$ has value 2?

(b) **(15 points)** Let A denote the event that there are 2 arrivals in the time interval (0, 6], and B the event that there are 2 arrivals in the time interval (3, 9]. Given that the event AB occurred, find the *conditional probability* that there was exactly one arrival in (3, 6].

5. **(32 points)** Let **X** denote the lifetime of a system.
If hypothesis H_0 is true, the hazard rate of **X** is given by $h(t) = 1$, $0 < t < \infty$.
If hypothesis H_1 is true, the hazard rate of **X** is given by $h(t) = 2$, $0 < t < \infty$.
The system failed at time T, that is, **X** took on value T on this trial of the experiment. The decision rule chooses H_i if the observed value T of **X** exceeds θ and H_{1-i} otherwise.

$$T \underset{H_{1-i}}{\overset{H_i}{\gtrless}} \theta$$

(a) **(5 points)** If this is the maximum-likelihood decision rule, what are the values of i and θ?

(b) **(12 points)** Find the false-alarm probability P_{FA} and the missed-detection probability P_{MD} of the maximum-likelihood decision rule.
Now assume that $P(H_0) = 0.6$ and $P(H_1) = 0.4$.

(c) **(5 points)** For what value of i and θ is the decision rule the minimum-error-probability (MEP) or Bayes' decision rule?

(d) **(10 points)** For what values of $\pi_0 = P(H_0)$, $0 < \pi_0 < 1$, does the MEP decision rule *always* choose H_0 regardless of the observed value of **X**?
For what values of $\pi_0 = P(H_0)$, $0 < \pi_0 < 1$, does the MEP decision rule *always* choose H_1 regardless of the observed value of **X**?