ECE 313
FIRST HOUR EXAMINATION
Monday October 1, 2001
11:00 a.m. — 11:50 a.m.

1. **(30 points)** Let A, B, and C denote three events defined on a sample space Ω, and suppose that $P(A) = 0.3$, $P(B) = 0.3$, $P(C) = 0.5$, and $P(A \cap B^c) = P(A^c \cap B^c \cap C) = 0.2$.

Find $P(A \cap B)$, $P(A^c \cap B)$, $P((A \quad B \quad C)^c)$, $P(B \cap C^c)$ and $P(C^c \mid (A^c \cap B^c))$.
If any of these probabilities cannot be determined from the given data, check the corresponding box and leave the rest of answer area blank.

2. **(20 points)** Events A, B, C, D, and E are defined on a sample space Ω.

(a) **(8 points)** If $P(A) = \frac{3}{5}$, $P(A \quad B) = \frac{4}{5}$, and $P(A|B) = \frac{1}{2}$, find P(B).

(b) **(12 points)** Suppose that C and D are *disjoint* events and that $P(C) = 2 \cdot P(D)$, $P(E|C) = \frac{3}{7}$ and $P(E|D) = \frac{2}{7}$. Find the value of $P(E|C \quad D)$.

3. **(35 points)** Fred suggests that he and Wilma play a game in which they will take turns tossing a fair coin; the first one to toss a Head wins. Fred proposes that he will toss first. Wilma agrees to this, but, having taken ECE 313, she knows that she is at an disadvantage. So, she demands that in succeeding games, the *loser* of the previous game gets to toss first.

(a) **(6 points)** *Compute* the probability that Fred wins the first game. You may have the answer written down on your sheet of notes, but I want to see if you can *derive* the answer from first principles.

(b) **(8 points)** What is the probability that Wilma wins the second game?

(c) **(6 points)** What is the probability that Wilma won the first game given that she won the second game?

(d) **(10 points)** The loser of each game pays the winner $1. Let **X** denote Wilma's winnings after two games. (Remember that **X** will have negative value if Wilma loses money.)

What is the probability mass function (pmf) $p_X(u)$ of **X**?

(e) **(5 points)** What is the mean value of **X**?

4. **(15 points)** Eight people hold reservations for travel in a 5-passenger limousine from Champaign to St. Louis. The number of persons who actually show up to travel can be modeled as a binomial random variable **X** with parameters $(8, \frac{1}{2})$. Note: $2^8 = 256$.

If more than 5 people show up, only the first 5 get to go, and the rest are left behind. What is the average number of passengers who are left behind?