University of Illinois at Urbana-Champaign

ECE 313: Probability with Engineering Applications

Fall 2000
Exam II

November 6, 2000

Name: _____

☐ Section C, 10:00 MWF ☐ Section D, 11:00 MWF

- You have **60 Minutes** for this exam.
- This exam is closed book; however, you may consult both sides of your $8.5'' \times 11''$ sheet of notes.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- It is *not necessary* to convert answers that are simple common fractions into decimal fractions, but you should simplify the fractions by canceling common factors from the numerator and denominator (e.g., write 5/16 instead of 10/32).
- Note that the questions are not weighted equally. Budget your time accordingly and do not work too long on any one problem.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. _____ (30 pts.)
2. _____ (30 pts.)
3. _____ (20 pts.)
4. _____ (20 pts.)

Total: _____(100 pts.)

Problem 1 *(30 points)* Let X be a continuous random variable whose pdf is given by

$$f_X(u) = \begin{cases} a + (b-a)u & : \quad 0 < u \leq 1 \\ 0 & : \quad \text{otherwise} \end{cases} .$$

If $E[X] = 2/3$, what is $P\{X < 1/2\}$? If the answer cannot be determined, check the box on the left, and leave the right hand box blank.

☐ Answer cannot be determined	$P\{X < 1/2\} =$

Problem 2 *(30 points)* Wolff, a CNN news correspondent who covered the Olympics in Sydney, has been on the road for so long that he can't remember where he is. He knows that he must be in either Atlanta or Sydney, and he knows the following likelihoods about the weather in Atlanta and Sydney at this time of year:

Likelihood Matrix	Weather in city X		
	Sunny	Rain	Snow
Atlanta	0.24	0.52	0.24
Sydney	0.48	0.32	0.20

Wolff would like to use this information to make a decision about his current location.

Part (a) 5 pts. How many possible decision rules exist for this problem?

Number of decision rules =

Part (b) 5 pts. Determine the the maximum likelihood decision rule and write it in the space below.

Problem 2 *(cont)*

Part (c) 5 pts. By looking at his travel calendar, Wolff estimates that the prior probability of his being in Atlanta on any given day is 3/4. He thus concludes that the prior probability that he is in Sydney is 1/4. Using this information, fill in the joint probability matrix below. For your convenience, the likelihood matrix from part (a) is reproduced below.

Likelihood Matrix	Weather in city X		
	Sunny	Rain	Snow
Atlanta	0.24	0.52	0.24
Sydney	0.48	0.32	0.20

	Weather in city X		
	Sunny	Rain	Snow
Atlanta			
Sydney			

Part (d) 5 pts. Assuming the same prior probabilities as in Part (c), what is the probability that it is a sunny day in Wolff's location?

P{Sunny} =

Part (e) 5 pts. Assuming the same prior probabilities as in Part (c), determine the the Bayes decision rule and write it in the space below.

Part (f) 5 pts. Assuming the same prior probabilities as in Part (c), what is probability of error associated with the *maximum likelihood* decision rule?

P{ error } =

Problem 3 *(20 points)* Let X be a Gaussian random variable that denotes temperature, with $\mu = 60$, $\sigma^2 = 400$.

Part (a) 5 pts. What is probability that the temperature is below zero?

$\mathrm{P}\{X < 0\} =$

Part (b) 5 pts. What is probablility that the temperature is above 100?

$\mathrm{P}\{X > 100\} =$

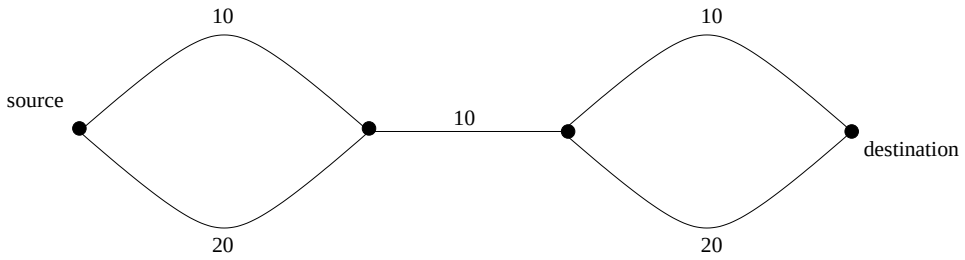
Part (c) 5 pts. What is probablility that the temperature is between 50 and 70 degrees?

$\mathrm{P}\{50 \leq X \leq 70\} =$

Part (d) 5 pts. What is probablility that the temperature is between 50 and 60 degrees?

$\mathrm{P}\{50 \leq X \leq 60\} =$

Problem 4 *(20 points)* Consider the following network, with link capacities (in packets) as shown.



Assume that each link fails independently with probability $p = 1/2$.

Part (a) 10 pts. What is the probability that a message can be sent successfully from the *source* to the *destination*?

Part (b) 6 pts. Let X denote the number of packets that can be sent from the *source* to the *destination*. Determine the pmf for X.

Part (c) 4 pts. Determine the expected value of X.