Assigned: Wednesday, August 25
Due: Wednesday, September 1
Reading: Ross, Chapter 1.1–1.5, Chapter 2.1–2.5 and 2.7
Noncredit Exercises: (Do not turn these in) Ross, p. 16: 1–5, 7, 9; p. 59-60: 3, 4, 9, 10, 11-14; pp. 61-64: 1–3, 6, 7, 10, 11, 12, 16.

Problems: These problems are based entirely on material covered in the *prerequisites* to this course. You should have mastered this stuff already, but may need to review the material one more time before starting the course. Think of this problem set as a diagnostic aid: if you cannot solve *all* these problems correctly, you will have difficulty in comprehending the material in the latter half of this course. It is not in your best interest to discover *after* the drop date that you really don't understand calculus as well as you thought you did, and that consequently you are in some danger of failing this course.

Do not use Mathematica or Matlab or a calculator etc. to do these problems except when you are specifically asked to do so.

1.(a) Does the commutative law of addition: a + b = b + a imply that – 4 + 1 equals 1 – 4?

(b) Determine whether -2^2+1 equals $1-2^2$ and -2^3+2 equals $2-2^3$ using

- **(i)** ordinary grade–school arithmetic.
- **(ii)** your calculator.
- **(iii)** the Microsoft spreadsheet program Excel. (Enter the four formulas = –2^2+1, = 1–2^2, = –2^3+2, and = 2–2^3 into different cells in the spreadsheet)

(c) True or False? The solutions to the quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{2c}{-b \pm \sqrt{b^2 - 4ac}}$$

2. The angles in this problem are expressed in **degrees** and **not in** the **radians** more commonly used in mathematical circles.

(a) Use your calculator to evaluate cot(10˚)cot(30˚)cot(50˚)cot(70˚) *without* ***writing*** *down intermediate results such as the values of* cot(10˚), cot(30˚), *etc and re-entering the numbers into your calculator.* If your calculator cannot be used in this fashion, you are urged to replace it with a more sophisticated machine.

(b) If your calculator's arithmetic unit is designed in accordance with the IEEE Standard for floating-point arithmetic, you should have obtained exactly 3 as the answer to part (a). Does it surprise you that cot(10˚)cot(30˚)cot(50˚)cot(70˚) = 3? Find four **other** integers a, b, c, and d such that $0 < a < b < c < d < 90$ and cot(a˚)cot(b˚)cot(c˚)cot(d˚) is an integer. You will get 500% extra credit on this problem if cot(a˚)cot(b˚)cot(c˚)cot(d˚) = 2.

(c) Find the value of $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}}$.

3. In **this** problem, all **angles** are expressed in **radians**.

(a) Use your calculator to evaluate $\sqrt{52}\cos(3^{-1}\arctan(18\sqrt{3}/35))$.

(b) Use your calculator to evaluate $\frac{1}{[\sin x]^2} - \frac{1}{x^2}$ for *small* values of x. Does the function seem to approach a limit, and if so, what is the limit? Now, use what you have learned in calculus to find $\lim_{x \to 0} \frac{1}{[\sin x]^2} - \frac{1}{x^2}$ analytically. (Hint: the answer is not 0, or 1, or ∞)

(c) Find the maxima of $f(x) = x^{25}(1.0001)^{-x}$ for $x > 0$. (If you have a graphing calculator, try it on this problem; otherwise just use standard calculus methods)

4.(a) What is the value of $\int_{-2}^{1} |x|\, dx$? the value of $\int_{-2}^{1} x(1-x)^{19}\, dx$?

(b) Prove or disprove: there exists a function f(x) satisfying **both** of the following two conditions:

(i) $f(x) \geq 0$ for all real numbers x in the range $-2 \leq x \leq 1$,

(ii) $\int_{-2}^{1} f(x)dx < 0$. (Hint: Does either function of part (a) satisfy both conditions?)

(c) Let $\frac{d}{dx}f(x) = g(x)$ for all x, $-\infty < x < \infty$. Which of the following statements are true for all x, $-\infty < x < \infty$? Note: C denotes an arbitrary constant.

(i) $\frac{d}{dx}f(-x) = g(-x)$. (ii) $\frac{d}{dx}f(x^2) = 2x\, g(x^2)$. (iii) $\frac{d}{dx}\exp(f(x^2)) = \exp(f(x^2))\, g(x^2)$.

(iv) $\int g(-x)dx = -f(-x) + C$. (v) $\int g(x^2)dx = f(x^2)/(2x) + C$. (vi) $\int \frac{g(x)}{f(x)}dx = \ln(f(x)) + C$

(d) Evaluate $\int_{1}^{\infty} x \bullet \exp(-x^2/2)dx$. (Hint: What is $\frac{d}{dx}\exp(-x^2/2)$?)

5.(a) What is the derivative of arctan(x)? (You can look up the answer if you like!)

(b) I denotes the value of the integral $\int_{-1}^{1} \frac{2}{1+x^2}dx$. Use the result of part (a) to show that I = π.

(c) J denotes the value of the integral $\int_{-1}^{1} \frac{2}{1+y^2}dy$. State True or False: I equals J.

(d) Make the substitution y = 1/x in the integral of part (b) and simplify the integrand to show that $I = \int_{-1}^{1} \frac{2}{1+x^2}dx = \int_{-1}^{1} \frac{-2}{1+y^2}dy = -J$? Does this contradict your answer to part (c)?

(e) I can equal **both** J **and** –J if and only if I = J = 0. Since you showed in part (b) that I = π, does this mean π = 0? (Such a result would greatly simplify a **lot** of engineering math!)

6. Let ABCD denote a trapezium in which sides AB and CD are parallel and side AB is shorter than side CD. Extend sides CB and DA to meet at E. Now, use the formula "Area of triangle = (1/2) × (length of base) × height" to show that the area of a trapezium is "(1/2) × (sum of the lengths of the parallel sides) × (distance between the parallel sides)."

7.(a) Let
$$f(x,y) = \begin{cases} x, & 0 < x \leq 1/2,\ x \leq y \leq 1-x, \\ 1-x, & 1/2 < x \leq 1,\ 1-x \leq y \leq x, \\ y, & 0 < y \leq 1/2,\ y < x < 1-y, \\ 1-y, & 1/2 < y \leq 1,\ y < x < 1-y, \\ 0, & \text{elsewhere.} \end{cases}$$

(i) Sketch the function g(x) = f(x,0.25) and find the area under this curve.

(ii) Compute the integral of f(x,y) over the entire plane.

(b) Compute the integral of $(x^2 + y^2)^{-2}$ over the region $\{(x,y) : x^2 + y^2 > 2\}$.