University of Illinois	**First Hour Exam**	**ECE 313 Fall 1999**

1. An urn contains n red and n black balls. Two balls are drawn at in succession from the urn. The first ball is not replaced in the urn before the second is drawn.

For i = 1, 2, let R_i denote the event that the i-th ball drawn is red

and $B_i = R_i^c$ denote the event that the i-th ball drawn is black.

(a) Find $P\{(R_1 \quad R_2) \cap (B_1 \quad B_2)\}$ and $P\{B_1 \mid (R_1 \quad R_2)$.

(b) Mark one box for each of the three statements. You do not need to justify your answers.

(i) R_1 and R_2 are disjoint (i.e. mutually exclusive) events ☐ TRUE ☐ FALSE

(ii) R_1 and R_2 are independent events ☐ TRUE ☐ FALSE

(iii) $P(R_1) = P(R_2)$ ☐ TRUE ☐ FALSE

2. A digital communication channel has inputs 0 and 1. The output signal is quantized as High or Low. It is known that

$$P\{\text{Low} \mid \text{input} = 0\} = P\{\text{High} \mid \text{input} = 1\} = \alpha$$

and that $\Pi_0 = P\{\text{input} = 0\} = \frac{7}{8}$, $\Pi_1 = P\{\text{input} = 1\} = \frac{1}{8}$

The receiver observes the output of the channel (High or Low as the case may be) and decides whether a 0 or a 1 was being transmitted

(a) Find the maximum-likelihood (ML) decision rule and $P_{e,ML}$, the average error probability of the ML decision rule.

(b) Find the Bayes' (that is, the maximum *a posteriori* probability (MAP) or minimum-error-probability) decision rule, and $P_{e,MAP}$, the average error probability of the MAP decision rule

(c) It is known that the ML decision rule and the MAP decision rule are identical if $\Pi_0 = \frac{1}{2}$.

Find all **other** values of Π_0 for which the ML and MAP decision rules are identical.

3. Let A and B denote events defined on a sample space, and suppose that

$P(A) = \frac{3}{5}$, $P(B) = \frac{3}{5}$, and that $P(B|A) = \frac{2}{3}$. Find $P(A|B)$, $P(A^c \quad B^c)$, and $P(B^c|A^c)$.

If the numerical value of any of these probabilities cannot be determined from the given information, check the corresponding little square on the left of the answer box, and leave the right side blank.

☐ $P(A\|B)$ cannot be computed	$P(A\|B) =$
☐ $P(A^c \quad B^c)$ cannot be computed	$P(A^c \quad B^c) =$
☐ $P(B^c\|A^c)$ cannot be computed	$P(B^c\|A^c) =$

4. You are playing in a three-set tennis match which, as usual, will end as soon as either you or your opponent wins two sets. You are a good player, but sadly out of shape, and estimate that the chances of your winning the first, second, and third sets are respectively $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{3}$, and that these events are independent.

(a) What is the probability that you win the match?

(b) Let **X** denote the number of the first set that you win. Assume that **X** has value 0 if you are thrashed in straight sets (i.e. do not win a set at all). Find E[**X**].

(c) Let **Y** denote the number of sets played in the match. Find E[**Y**].