University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 313
Hour Exam # 2

Monday, November 16, 1998
7:00 p.m. — 8:00 p.m.
Room 112 Chemistry Annex Building

Name ____________________

University ID Number ____________________

Signature ____________________

Section:
- ☐ B 9 MWF
- ☐ D 11 MWF

INSTRUCTIONS

This exam is closed book and closed notes, except that one $8.5'' \times 11''$ sheet of notes (both sides) is allowed. Calculators will be provided, so you may not use your own calculators, laptop computers, tables of integrals, etc.

The exam consists of 4 problems worth a total of 120 points. Of these, 20 points in Problem 4 are **extra-credit points**. Scores in excess of 100 points will compensate for other Exam and Homework scores. Write your answers in the spaces provided. Show *all* your work; if you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem.

Grading

1.	26 points	______
2.	22 points	______
3.	32 points	______
4.	20 points	______
extra:	20 points	______
	TOTAL	______

Problem 1 (26 points)

Bubba rolls a fair 6-faced die till he rolls the number 6 for the second time. Let $\mathcal{S}$ be a random variable that denotes the number of times he rolls the die.

a. Find $P\{\mathcal{S} > 6\}$.

$P\{\mathcal{S} > 6\} \ =$

b. Find the probability mass function of $\mathcal{S}$, i.e., find $p_{\mathcal{S}}(k) = P\{\mathcal{S} = k\}$. *Specify* $p_{\mathcal{S}}(k)$ *for all values of* k.

$P\{\mathcal{S} = k\} \ = \begin{cases} \\ \end{cases}$

c. If Bubba gets 2 sixes within 10 rolls, he gets $\dfrac{10}{\mathcal{S}-1}$ points, otherwise he gets 0 points.

For example, if he gets 2 sixes in 7 rolls, then he gets $10/6$ points; on the other hand, if he gets 2 sixes in 11 rolls, he gets 0 points.
Find the expected number of points that he gets.

$E[\text{number of points}] \ =$

Problem 2 (22 points)

X is a continuous random variable with probability density function given by $f_X(u) = e^{-\pi u^2}$, for all $u \in \mathbb{R}$. Define a new random variable Y by $Y = (1 - \sqrt{\pi} X)^2$.

a. Find the expected value of Y.

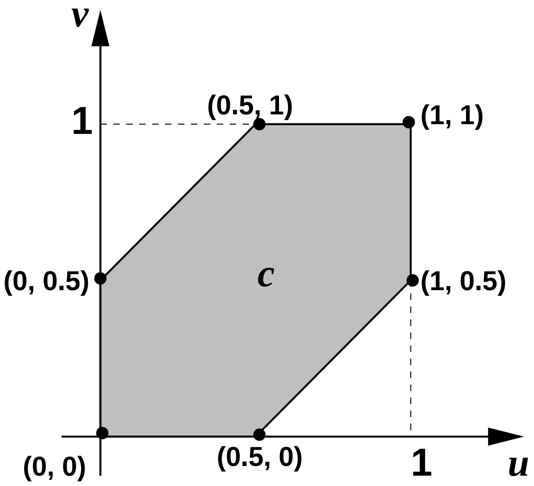
$$\boxed{E[Y] \;=\; \qquad\qquad}$$

b. What is the probability density function of Y, $f_Y(v)$? *Specify this for all* $v \in \mathbb{R}$.

$$\boxed{f_Y(v) \;=\; \begin{cases} & \\ & \\ & \end{cases}}$$

Problem 3 (32 points)

Random variables X and Y are jointly continuous with a joint probability density function that is a constant c over the hexagon shown in the figure below.



a. What is the value of c?

$c =$

b. Find the marginal probability density function $f_Y(a)$ (the marginal pdf of Y), for all $a \in \mathbb{R}$. *Specify the pdf for all values of* a.

$f_Y(a) = \Big\{$

c. Find the value of the joint cumulative distribution function at the point $\left(\dfrac{3}{4}, \dfrac{3}{4}\right)$.

$\boxed{F_{X,Y}\left(\tfrac{3}{4}, \tfrac{3}{4}\right) \; = \qquad\qquad}$

d. Calculate the probability that $\left(X - \dfrac{1}{2}\right)^2 + \left(Y - \dfrac{1}{2}\right)^2 \leq \dfrac{1}{4}$.

$\boxed{P\left\{\left(X - \tfrac{1}{2}\right)^2 + \left(Y - \tfrac{1}{2}\right)^2 \leq \tfrac{1}{4}\right\} \; = \qquad\qquad}$

Problem 4 (20 points) The number of arrivals at a bank is a Poisson random variable with an average rate of 20 customers/hour.

a. Find the probability that 5 customers enter the bank in a 15 minute interval.

$P(5 \text{ people in a 15 minute interval}) =$

b. Given that 20 customers entered the bank in 1 hour, what is the probability that 5 of them entered in the **first** 15 minutes?

$P(5 \text{ in first 15 minutes} \mid 20 \text{ in one hour}) =$

c. **Extra Credit [20 pts]**: Given that **only** 1 person entered the bank in one hour, find the probability density function of her **time of entry**, denoted by the random variable $\mathcal{T}$.

$$f_{\mathcal{T}}(u) = \begin{cases} & \\ & \end{cases}$$