University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 313
Hour Exam # 1

Monday, October 12, 1998
7:00 p.m. — 8:00 p.m.
Room 112 Chemistry Annex Building

Name ______________________________

University ID Number ______________________________

Signature ______________________________

Section:

- ☐ B 9 MWF
- ☐ D 11 MWF

INSTRUCTIONS

This exam is closed book and closed notes, except that one 8.5″ × 11″ sheet of notes (both sides) is allowed. Calculators, laptop computers, tables of integrals, etc. may not be used.

The exam consists of 4 problems worth a total of 100 points. Write your answers in the spaces provided. Show *all* your work, except in the true/false problem. If you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem.

Grading

1.	24 points	________
2.	24 points	________
3.	32 points	________
4.	20 points	________
	TOTAL	________

Problem 1 (24 points)

In the following statements, events A, B, and C have nonzero probabilities $P(A)$, $P(B)$, and $P(C)$; the events are arbitrary, unless a specific condition is specified in the statement. The random variable X is also arbitrary, unless otherwise specified in the statement. The function $F_X(u)$ is the cumulative distribution function (CDF) of X.
Answer True if the statement is true for **all** such events A, B, C, and random variables X. Otherwise, answer False.

True	False	
☐	☐	If A and B are independent, then AB^c and A^cB are independent.
☐	☐	if the events A, B, C are disjoint and $P(A) + P(B) + P(C) = 1$, then exactly five of the eight regions in the Karnaugh map for A, B, C have zero probability
☐	☐	$P\Big((A \oplus B)\vert A\Big) = P(B^c)$
☐	☐	If $P(A\vert B) = P(B\vert A)$, then $P(A) = P(B)$
☐	☐	$P(AB) \leq \sqrt{P(A)P(B)}$
☐	☐	If A and B are disjoint, then $P(A\vert B) + P(A\vert B^c) = 1$
☐	☐	If $P\{X = \pi\} = P\{X = -\pi\} = \frac{1}{2}$, then $\sin(X)$ is a discrete random variable
☐	☐	If $a < b$ then $F_X(a) < F_X(b)$

Check the appropriate box for each of the eight statements above. **No justification is required**. However, so as not to give points for random guessing, you will receive +3 points for correct answer, 0 points for no answer, and −2 points for wrong answer.

Problem 2 (24 points)

Dilbert has 3 coins in his pocket, 2 of which are fair, and 1 which is biased. The probability that a coin picked at random from his pocket will land Tails when it is tossed is 7/12.

a. What is the probability that the biased coin will fall Tails on a given toss?

$$\boxed{P(\text{Tails}) \;=\; \qquad}$$

b. If Dilbert picks **two** coins out of his pocket what is the probability that he did **not** pick the biased coin?

$$\boxed{P\Big((\text{Biased coin})^c\Big) \;=\; \qquad}$$

c. If he picks **two** coins out of his pocket, tosses each one **once**, and observes a Head and a Tail, what is the (conditional) probability that he did **not** pick the biased coin?

$$\boxed{P\Big((\text{Biased coin})^c | 1 \text{ Head } \& \text{ 1 Tail}\Big) \;=\; \qquad}$$

Problem 3 (32 points)

Toejam Inc., a new shoe store, has a special opening offer this week. They have 13 different pairs of shoes in a barrel at the door. A customer pays \$1 for the chance of picking out one shoe at random from the barrel. You and your friend walk into the store and decide to pick **two** shoes **each** in the hopes of getting a matching pair.

a. If you pick first, what is the probability of your getting a matching pair of shoes?

$P(\text{matching pair}) =$

b. Your friend picks next. What is the probability that she gets a matching pair of shoes?

$P(\text{matching pair}) =$

c. What is the probability that there is at least one matching pair of shoes in the 4 that have been picked?

$$P(\text{at least 1 matching pair}) =$$

d. If N is a random variable denoting the number of matching pairs in the 4 shoes that have been picked, find the probability mass function (pmf) of N, i.e., find $P\{N = u\}$ for all $u \in \mathbb{R}$.
To obtain full credit you have to specify the pmf of N for **all** *real numbers $u \in \mathbb{R}$.*

$$p_N(u) = \left\{ \begin{array}{l} \\ \\ \end{array} \right.$$

Problem 4 (20 points)

Y is a **continuous** random variable. Its cumulative distribution function (CDF) is defined as follows:

$$F_Y(u) = \begin{cases} 0, & u < -4 \\ \dfrac{u+4}{6}, & -4 \le u < -2 \\ \alpha\, u^2 + \beta\, u + \dfrac{1}{2}, & -2 \le u < 2 \\ 1, & u \ge 2 \end{cases}$$

a. What is the probability that $|Y| \ge 3$?

$P\{|Y| \ge 3\} \;=$

b. Find the values of the real numbers α and β.

$\alpha \;=$

$\beta \;=$

c. Calculate the probability $P\{Y^2 - 1 < 0\}$.

$P\{Y^2 - 1 < 0\} \;=$