ECE 313: Problem Set #7

Assigned: Wednesday, October 7, 1998
Due: Friday, October 16, 1998

1. The amount of bread (in hundreds of pounds) that a bakery sells in a day is a random variable X with probability density function

$$f_X(u) = \begin{cases} cu & 0 \le u < 3 \\ c(6-u) & 3 \le u < 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the constant c.

(b) Compute the cdf $F_X(u)$ of X.

(c) Show that the function $F_X(u)$ computed in part **(b)** satisfies all the four properties of cdfs given in [Ross, p. 132].

(d) What is the probability that the number of pounds of bread sold in a single day will be (i) more than 300 pounds? (ii) between 150 and 900 pounds?

(e) If A and B are the events in (i) and (ii), respectively, are these events independent?

2. A number is randomly chosen (that is, with uniform distribution) from the interval $(0, 1)$. What is the probability that

(a) its first decimal digit will be a 1?

(b) its second decimal digit will be a 2?

(c) the first decimal digit of its square root will be a 3?

3. Suppose that the duration in minutes of long-distance telephone conversations follows an exponential probability density function

$$f_X(u) = \begin{cases} \frac{1}{5}e^{-u/5} & u > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that the duration of the conversation

(a) will exceed 5 minutes?

(b) will be less than 6 minutes?

(c) will be between 5 and 6 minutes?

(d) will be less than 6 minutes, given that it was greater than 5 minutes?

4. $\mathbf{X}$ is a geometric random variable with parameter $1/2$, and $\mathbf{Y} = \sin(\pi\mathbf{X}/2)$. Is $\mathbf{Y}$ discrete, continuous, or mixed? Find the CDF and pdf/pmf of $\mathbf{Y}$.