University of Illinois Fall 1998

ECE 313: Problem Set #6

Assigned: Wednesday, September 30, 1998
Due: Wednesday, October 7, 1998

1. In successive rolls of a pair of fair dice, what is the probability of rolling "seven" twice (not necessarily on consecutive rolls), before rolling an even number six times (not necessarily on consecutive rolls)?

2. Let the random variable $\mathcal{I}_D$ denote the indicator function of an event D. That is,

$$\mathcal{I}_D(\omega) = \begin{cases} 1 & \text{if } \omega \in D \\ 0 & \text{otherwise} \end{cases}$$

Let A, B, and C denote independent events, with $P(A) = P(B) = P(C) = 0.5$. Define the random variable X by $X(\omega) = \mathcal{I}_A(\omega) + 2\mathcal{I}_B(\omega) - \mathcal{I}_C(\omega)$.

(a) What are the values taken on by the random variable X?

(b) Find the cumulative probability distribution function $F_X(u)$ and the probability mass function $p_X(u)$ of the random variable X. Be very careful in specifying the values of $F_X(u)$ at points where the function is discontinuous.

3. Let $\Omega = \{1, 2, 3, 4, 5\}$. Choose a single probability measure on Ω so that you are able to complete all of the following tasks.

(a) Define a random variable X on Ω that takes the values $1, 2, 3, 4, 5$ with probabilities $0.1, 0.1, 0.2, 0.2, 0.4$ respectively.

(b) Define another random variable Y on Ω that takes the values $\sqrt{2}, \sqrt{3}, \pi$ with probabilities $0.2, 0.3, 0.5$ respectively.

(c) Consider the random variable $Z = XY$. What is the set of values $\{z_1, z_2, \ldots, z_n\}$ of Z. For each z_i in this set, what is the probability that $Z = z_i$?

Hint: the answers to (**a**) and (**b**) are not unique.

4. Which of the following are valid cumulative probability distribution functions? For those that are not valid cdfs, state at least one property of the cdf which is not satisfied. For those that are valid cdfs, compute $P\{|X| > 0.5\}$.

(a) $$F_X(u) = \begin{cases} 0 & u < 1 \\ 2u - u^2 & 1 \le u \le 2 \\ 1 & u > 2 \end{cases}$$

(b) $$F_X(u) = \begin{cases} 0.5e^{2u} & u < 0 \\ 1 - 0.25e^{-3u} & u \ge 0 \end{cases}$$

(c) $$F_X(u) = \begin{cases} 0.5e^{2u} & u \le 0 \\ 1 - 0.25e^{-3u} & u > 0 \end{cases}$$

5. Ross, p. **175**, Problem **17**.

6. Calvin's drawer has 10 pairs of socks in it. He needs to pick out a pair of socks that match, to wear to school (otherwise Ms. Wormwood will probably send him to detention). He starts picking out socks at random, one at a time without putting them back in, till he gets a pair that match. If X is a random variable that represents the number of socks he picks out, find the pmf of X, i.e. find $P\{X = i\},\ i = 1, 2, \ldots, 20$.
If he changes his strategy and picks a random **pair** of socks at a time, what would the pmf of the random variable Y be, where Y represents the number of **pairs** that he picks out.

7. **[15 pts]**

- **(a)** Ross, p. **188**, Problem **28 (a)**.
- **(b)** Ross, p. **190**, Problem **10**.

8. **[Extra Credit: 20pts]** Let A and B be independent events. Show that the probability of at least one of the following events, AB, A^cB^c, and $A \oplus B$ is greater than or equal to $4/9$.