University of Illinois Fall 1998

ECE 313: Problem Set #4

Assigned: September 16, 1998
Due: September 23, 1998
Reading: Ross, Chapter 3

1. The probability that an archer hits the target when it is windy is 0.4, and when it is not windy the probability of a hit is 0.7. On any shot, the probability of a gust of wind is 0.3. Find the probability that

(**a**) the target is hit with a shot

(**b**) there was a gust of wind assuming the target was missed.

2. (**a**) Consider 3 urns. Urn E contains 3 white and 3 red balls; urn F contains 8 white and 4 red balls; and urn G contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn E was white, given that exactly 2 white balls were selected?

(**b**) Same problem statement as above: If a genie blindfolds you and then hands you a ball picked at random from one of the urns, what is the probability that it came from urn E, given that it is white?

3. Ternary Channel: There are three equally probable transmitted signals in a communication system: $s_1, s_2,$ and s_3. The transmission is corrupted by noise, causing the received signal to be changed according to the following table of conditional probabilities:

		Receive j		
		s_1	s_2	s_3
Send i	s_1	0.8	0.1	0.1
	s_2	0.05	0.9	0.05
	s_3	0.02	0.08	0.9

For example: if s_1 is sent, the probability of receiving s_3 is 0.1. The entries list the probability of s_j received, given that s_i is sent. Compute the probabilities that s_1, s_2, s_3 are received. Compute the probabilities $P[s_i \text{ sent} \mid s_j \text{ received}]$ for $i, j = 1, 2, 3$.

4. (**a**) Residents (citizens and aliens) of the planet Narg commute aerially in vehicles called narbees. The probability of a randomly chosen airborne narbee being involved in an accident is 0.59 (it's a good thing they're all rubber). If a sober Nargian is behind the wheel, the probability of it being involved in an accident is 0.2. However, if an inebriated Nargian (they imbibe copious amounts of this great alcoholic beverage called Narniken) is flying, the probability of it being involved in an accident is 0.8. The matter is complicated by the presence of aliens, who get hopelessly confused by Narg's "Fuzzy Traffic Code", and have a probability of 0.5 of being involved in an accident. If a randomly chosen Nargian is twice as likely to be drunk as sober, what is the probability that a randomly chosen narbee driver is an alien?

(**b**) If a narbee flight did **not** result in an accident, what is the probability that it was piloted by a drunk Nargian?

5. Ross, problem 18, p. 106.

6. Ross, problem 56, p. 112.

7. **Dilbert's Ruin?** Dilbert-Geek and Wally-Loser are contestants on Monty Hall's "Let's Make a Deal" (as usual one of the curtains conceals a prize, and the other two, junk). We figured out in class how Dilbert could negotiate Monty's "new-and-improved-deal" to his advantage, and we can safely assume that Dilbert-Geek figures this out too.
In a follow-up to this game, Monty calls Wally down from the audience to join Dilbert and asks each of them to pick a curtain. Monty opens one of the curtains *that was picked* revealing junk behind it and sends that person back to the audience. The other player is now offered the choice of staying with his original choice or switching to the remaining curtain.

   (**a**) Compute the probability of the remaining player winning under both strategies — staying put OR switching. Is this the same as before? So what is his best strategy?

   (**b**) What is the probability that Dilbert wins the prize? What is the probability that Wally wins the prize?

   Note: Monty always opens one of the two curtains picked by his contestants. In some cases, one of the two curtains conceals the prize and so he is forced to open the other. If neither curtain conceals the prize then he opens one at random.

   (**c**) **Extra credit [10 pts]:** Having seen that Dilbert is possessed of extraordinary analytical capabilities, Monty decides to put him to the ultimate test — the *4 curtain test*, on an adjoining stage. There are now *four* curtains, one concealing a prize and the other 3 concealing junk. Dilbert picks a curtain and Monty opens one of the curtains revealing junk (again, remember that Monty knows where the prize is), and offers Dilbert the "new improved deal" — Dilbert has the choice of sticking to his decision or switching. Whether he stays or switches, Monty opens *another* curtain once Dilbert has made his choice (so now he has opened 2 curtains, one of which could have been Dilbert's first choice, if he switched) and again offers him the "new improved deal". What should Dilbert's two decisions be (remember, at this stage there are two curtains that Monty has opened and Dilbert has two curtains left to choose between)? How does the problem generalize to n curtains and one prize?