

Assigned : Wednesday, September 9, 1998

Due : Wednesday, September 16, 1998

Reading : Ross, Chapter 2, Chapter 3.2

1. Let A, B, C denote three events defined on the same sample space. Prove that
 - (a) $(P(A) + P(B) + P(C)) / 3 \leq P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$.
 - (b) Bonferroni's inequality: $P(ABC) \geq P(A) + P(B) + P(C) - 2$.
 - (c) Generalized Bonferroni's inequality: Given n events A_1, A_2, \dots, A_n , use induction to show that

$$P(A_1 A_2 \dots A_n) \geq P(A_1) + P(A_2) + \dots P(A_n) - (n - 1).$$

2. Find $P(A \cup (B^c \cup C^c)^c)$ in each of the following cases:
 - (a) A, B, C are mutually exclusive events and $P(A) = 3/7$.
 - (b) $P(A) = 1/2, P(BC) = 1/3, P(AC) = 0$.
 - (c) $P(A^c B^c \cup A^c C^c) = 3/7$.
3. Ross Chapter 2, p. 60, Problem 43.
4. Ross Chapter 2, p. 61, Problem 56.
5.
 - (a) Compute the probability that a bridge hand (13 cards) is void (no card) in at least one suit (Spades, Clubs, Hearts and Diamonds).
 - (b) Compute the probability that a bridge hand has at least one card from each suit (Spades, Clubs, Hearts and Diamonds).
6. **Matching Problem [15 pts]:** A router receives 8 packets that need to be routed to stations A, B, C, D, E, F, G , and H , each station receiving exactly one packet. Due to a lightning strike at the router site, it cannot parse the header information in a packet and hence, with equal probability, sends a given packet to any one of the 8 stations.
 - (a) Find the probability that the router redirects all packets wrongly.
 - (b) Find the probability that the router gets at least 6 packets routed correctly.
 - (c) Given that Station A does *not* receive the correct packet, what is the probability that *none* of the stations receive their designated packets?

7. An ordinary deck of 52 cards is shuffled and then cards are overturned one at a time till the first ace appears. Given that the first ace is the 3^{rd} card to appear, what is the conditional probability that the card following it is (i) Ace of Spades (ii) Two of Clubs? How do the conditional probabilities change if the first ace is the 20^{th} card?
8. **[Extra Credit 10 pts]:** This is a modification of the above problem. A deck of cards is shuffled and then the cards are overturned one by one till the first ace is encountered. What is the probability that the next card is an Ace of Spades? What is the probability that the next card is a Two of Clubs?