Assigned: Wednesday, November 5, 1997
Due: Wednesday, November 12, 1997
Reading: Ross, Chapters 4 and 5
Reminder: Hour Exam II is scheduled for Monday November 17 from 7 p.m. to 8 p.m. in Room 228, Natural History Building

Suggested Noncredit Exercises: Ross, pp. 173-184: 2, 7, 13, 17, 20, 25, 37, 39, 41-45; pp. 184-188: 10-13; pp. 232-237: 1-8; pp. 237-241: 1, 8

Those using the 4th edition should try Ross pp. 177-178: 10-13; pp. 182-197: 2, 7, 13, 17, 20, 25, 37, 39, 41-45; pp. 239-243: 1, 8; pp. 243-248: 1-8

Problems:

1. Let **X** denote the number of hours that a student works on ECE 340 each week. It is known that **X** is a mixed random variable with cumulative probability distribution function $F_X(u)$ given by

$$F_X(u) = \begin{cases} 0, & u < 0, \\ (1+u)/8, & 0 \le u < 1, \\ 1/2, & 1 \le u < 2, \\ 1/2 + u/8, & 2 \le u < 4, \\ 1, & u \ge 4. \end{cases}$$

Find the probability that the student

(a) works for exactly 2 hours,
(b) works for more than 2 hours,
(c) works for less than 2 hours,
(d) works for exactly 3 hours,
(e) works for more than 1/2 but less than 3 hours,
(f) works for more than 2 hours given that the student works at all, i.e. given that $\{\mathbf{X} > 0\}$.

2. Let **X** denote a binomial random variable with parameters (N, p). What is the probability that **X** is an even integer? Remember that 0 is an even integer.
[Hint: What is $(x+y)^n + (x-y)^n$?]

3. In Problem 4(a) of Problem Set #6, let **X** denote the number of passengers who *don't show up* for the flight. Then, **X** is a binomial random variable with parameters (105, 0.1).

(a) *Express* the probability that everyone who shows up gets a seat in terms of the random variable **X**, e.g. in terms of the CDF or the pmf of **X**. We found the exact value of this probability already.

The binomial random variable **X** can be approximated by a Poisson random variable **Y** since the parameters (N, p) are such that N is large and p is small.

(b) What is the parameter λ for the random variable **Y**?
(c) *Express* the probability that everyone who shows up gets a seat in terms of the random variable **Y**, e.g. in terms of the CDF or the pmf of **Y**, and *evaluate* the expression numerically
(d) The numerical answer to part (c) is, of course, an approximation to the exact probability that we computed in Problem Set #6. What is the absolute error of the approximation? What is the **relative** error of the approximation, where the relative error is defined as

$$\frac{\text{approx value} - \text{true value}}{\text{true value}} \times 100\% \text{ ?}$$

4. Let **X** denote a Poisson random variable with unknown parameter λ. Suppose that the event $\{\mathbf{X} = k\}$ occurs.

(a) What is the maximum–likelihood estimate of λ? That is, what value of λ maximizes the probability of the observed event $\{\mathbf{X} = k\}$?

(b) Consider a binomial random variable **Y** with parameters (N, p) where the parameter p is unknown. If the event {**Y** = k} is observed (e.g. heads occurs k times on N tosses of a biased coin with P(Heads) = p), then we showed in class that $\hat{p} = k/N$ is the maximum–likelihood estimate of p. Since for large N and small p, the binomial random variable **Y** can be approximated by a Poisson random variable **X** with parameter $\lambda = Np$, it would seem reasonable that the maximum-likelihood estimate of λ would be $\hat{\lambda} = N\hat{p} = k$. Does your answer to part (a) give this result?

5. Which of the following are valid probability density functions? Assume that the functions are zero outside the ranges specified. For those which are not valid pdfs, state at least one property of pdfs which is not satisfied. Also, state whether there exists a constant C such that Cf(u) is a valid pdf even though f(u) is not.

(a) $f(u) = |u|$ for $|u| < 1$.
(b) $f(u) = 1 - |u|$ for $|u| < 1$.
(c) $f(u) = \ln u$ for $0 < u < 1$,
(d) $f(u) = \ln u$ for $0 < u < 2$. Hint: ln u can be integrated by parts
(e) $f(u) = 2u$ for $0 < u < 1$.
(f) $f(u) = (2/3)(u - 1)$ for $0 < u < 3$.
(g) $f(u) = \exp(-2u)$, $0 < u < \infty$,
(h) $f(u) = 4\exp(-2u) - \exp(-u)$, $0 < u < \infty$.

6. The random variable **X** has probability density function

$$f_{\mathbf{X}}(u) = \begin{cases} \alpha(1-u), & 0 < u < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $P\{6\mathbf{X}^2 > 5\mathbf{X} - 1\}$.
(b) Find $F_{\mathbf{X}}(u)$. Be sure to specify the value of $F_{\mathbf{X}}(u)$ for all u.