University of Illinois | **Problem Set #3** | **ECE 313**

Fall 1997

Assigned: Wednesday, September 17, 1997
Due: Wednesday, September 24, 1997
Reading: Ross, Chapter 3
Noncredit Exercises: (Do not turn these in) Ross pp. 104-117: 1, 2, 4-6, 10, 12, 16, 21, 31, 38, 39, 43-45; pp. 118-122: 4, 23; p. 123; 2,3
Those with the **4th** edition should try the following problems. Ross pp. 106-112: 3, 20; pp. 112-125: 1, 2, 4-6, 11, 13, 17, 19, 22, 23, 31, 39, 40, 43-45.

Problems:

1. Let A, B, C denote three events defined on a sample space. Show that

$$[P(A) + P(B) + P(C)]/3 \le P(A \quad B \quad C) \le P(A) + P(B) + P(C)$$

2. Find $P(A \quad (B^c \quad C^c)^c)$ in each of the following four cases:

(a) A, B, and C are mutually exclusive events and P(A) = 1/3.

(b) P(A) = 2P(BC) = 4P(ABC) = 1/2 **(c)** P(A) = 1/2, P(BC) = 1/3, and P(AC) = 0

(d) $P(A^c \cap (B^c \quad C^c)) = 0.6$

3. This problem on conditional probability has three ***unrelated*** parts.

(a) If $P(A|B) = 0.3$, $P(A^c|B^c) = 0.4$, and $P(B) = 0.7$, find $P(A|B^c)$, $P(A)$, and $P(B|A)$.

(b) If $P(E) = 1/4$, $P(F|E) =1/2$, and $P(E|F) = 1/3$, find $P(F)$.

(c) If $P(G) = P(H) = 2/3$, show that $P(G|H) \ge 1/2$.

4. Monty Hall, the host of the TV game show "Let's Make A Deal™", shows you three curtains. One curtain conceals a valuable prize, while the other two conceal junk. All three curtains are equally likely to conceal the prize. He offers you the following "deal": pick a curtain, and you can have whatever is behind it. When you pick a curtain, instead of giving you your just deserts, Monty (who knows where the prize is) opens one of the remaining curtains to show you that there is junk behind it, and offers the following "new, improved deal™" : you can either stick with your original choice, or switch to the remaining (unopened) curtain. Amidst the deafening roars of "Stand pat" and "Switch, you idiot" from the crowd, Monty points out that previously your chances of winning were 1/3. Now, since you know that the prize is behind one of the two unopened curtains, your chances of winning have increased to 1/2, and thus the new improved deal is indeed better. Use the theorem of total probability (Ross, Equation (3.1), Chapter 3) to determine

(a) the probability of winning if you always switch.

(b) the probability of winning if you would rather fight than switch.

(c) whether Monty is correct in asserting that if you choose randomly betweeen the two unopened curtains, you have a probability of winning of 1/2.
Note: Everybody knows that the rules of the game are that Monty always opens one of the two unchosen curtains and he always offers the "new improved deal," i.e. he never opens a curtain to reveal the prize (saying "Oops, you lose; return to your seat")

5. At the County Fair, you see a man sitting at a table and rapidly rolling a pea between three walnut shells. "Step right up, me bucko, and try your luck! The hand is quicker than the eye!" he says, and hides the pea under one of the shells. You have no idea which shell is covering the pea, but you point to one shell at random and bet that the pea is under it. The man picks up one of the shells that you didn't choose, and shows you that the pea is not underneath that shell. He asks if you would like to switch your bet to the other unchosen shell. Should you accept the offer? Why or why not? How does this game differ from the one analyzed in Problem 4?

6. The experiment consists of picking one team at random from the 280 teams in Division I of the NBAA (National Basketball Association of America) who play another Division I team on a given Saturday afternoon. Note that there are 140 games, and each game continues (with as many overtimes as necessary) until one team wins. If a team is leading at half time, the (conditional) probability that it wins its game is 0.7. Given that a team wins its game, the (conditional) probability that it led at half time is 0.6. What is the probability that the team picked was tied with its opponent at half time? How many games were tied at half time?