HW11 Solutions

ECE 310: Digital Signal Processing, Spring 2026 Version: **1.0**

Due Date: 4/24/26
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1

Solution:

(a) **Filter type: Low-pass filter.** The passband should end at the music's highest frequency (4 kHz) and the stopband should start at the noise's lowest frequency (8 kHz). Since we know the sampling frequency (24 kHz), we can use $\omega = \Omega T$ to determine the corresponding ranges in digital frequency:

- Passband: $0 \leq \omega \leq \frac{\pi}{3}$
- Stopband: $\frac{2\pi}{3} \leq \omega \leq \pi$

(b) **Filter type: Band-pass filter.** We want to isolate the second speaker (300–500 Hz) and remove the first (50–200 Hz) and third (600–800 Hz) speakers. Since we know the sampling frequency (2 kHz), we can use $\omega = \Omega T$ to determine the corresponding ranges in digital frequency:

- Lower stopband: $0 \leq \omega \leq \frac{\pi}{5}$
- Passband: $\frac{3\pi}{10} \leq \omega \leq \frac{\pi}{2}$
- Higher stopband: $\frac{3\pi}{5} \leq \omega \leq \pi$

Note that because our goal is to isolate the second speaker, we let the stopbands go to all the way to 0 and π. This is consistent with the definition of a band-pass filter.

Grading: *20 points, 10 points each*

- +10: Parts (a), (b)
  - -0: Correct answer.
  - -3 points: Minor mistake — correct filter type but incorrect cutoff frequency, or missed one frequency range.
  - -7 points: Major mistake — incorrect frequency range or filter type, but shows some understanding of filter behavior.
  - -10: Empty or invalid.

2

Solution:

(a) **Transfer function:** Taking the z-transform of the difference equation (assuming initial rest) and solving $H(z) = \frac{Y(z)}{X(z)}$, we get:

$$Y(z) = -\tfrac{1}{2}z^{-1}Y(z) + z^{-2}Y(z) + 2X(z) - \tfrac{1}{4}z^{-1}X(z) + z^{-2}X(z),$$
$$Y(z)\left(1 + \tfrac{1}{2}z^{-1} - z^{-2}\right) = X(z)\left(2 - \tfrac{1}{4}z^{-1} + z^{-2}\right).$$

Thus, the transfer function of the system is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - \frac{1}{4}z^{-1} + z^{-2}}{1 + \frac{1}{2}z^{-1} - z^{-2}}. \tag{1}$$

(b) Please look at Figure 1 below.

(c) Please look at Figure 1 below.

Grading: *30 points, 10 points each*

- +10: Part (a)
  - -0: Correct answer.
  - -3: Minor mistake — correct filter $H(z)$ but one or two coefficients are incorrect.
  - -7: Major mistake — incorrect $H(z)$ formula (e.g., $X(z)$ and $Y(z)$ reversed).
  - -10: Empty or invalid.
- +10: Parts (b), (c)
  - -0: Correct figures.
  - -1: Correct Direct Form structures but coefficients are incorrect, consistent with part (a).
  - -3: Minor mistake — correct Direct Forms but some coefficient signs are incorrect (e.g., $\pm$ errors).
  - -7: Major mistake — incorrect Direct Form structures but shows some understanding of the filters.
  - -10: Empty or invalid.

3

Solution:

(a)

The filter is $h[n] = \{1, -1\}$. The length is $N = 2$ (even). We check for symmetry: $h[0] = 1$ and $h[N-0-1] = h[1] = -1$. Since $h[0] = -h[1]$, the filter has **odd symmetry**. An even-length filter with odd symmetry is a Type-IV filter. Therefore, it **is a GLP filter**.

The frequency response is:

$$\begin{aligned}
H(\omega) &= 1 - e^{-j\omega} \\
&= e^{-j\frac{1}{2}\omega}\left(e^{j\frac{1}{2}\omega} - e^{-j\frac{1}{2}\omega}\right) \\
&= e^{-j\frac{1}{2}\omega}\left(2j\sin\left(\frac{1}{2}\omega\right)\right) \\
&= 2\sin\left(\frac{1}{2}\omega\right)e^{-j\frac{1}{2}\omega}e^{j\frac{\pi}{2}} \\
&= 2\sin\left(\frac{1}{2}\omega\right)e^{j\left(-\frac{1}{2}\omega+\frac{\pi}{2}\right)}
\end{aligned}$$

Comparing this to $H_d(\omega) = A(\omega)e^{j(-\alpha\omega+\beta)}$:

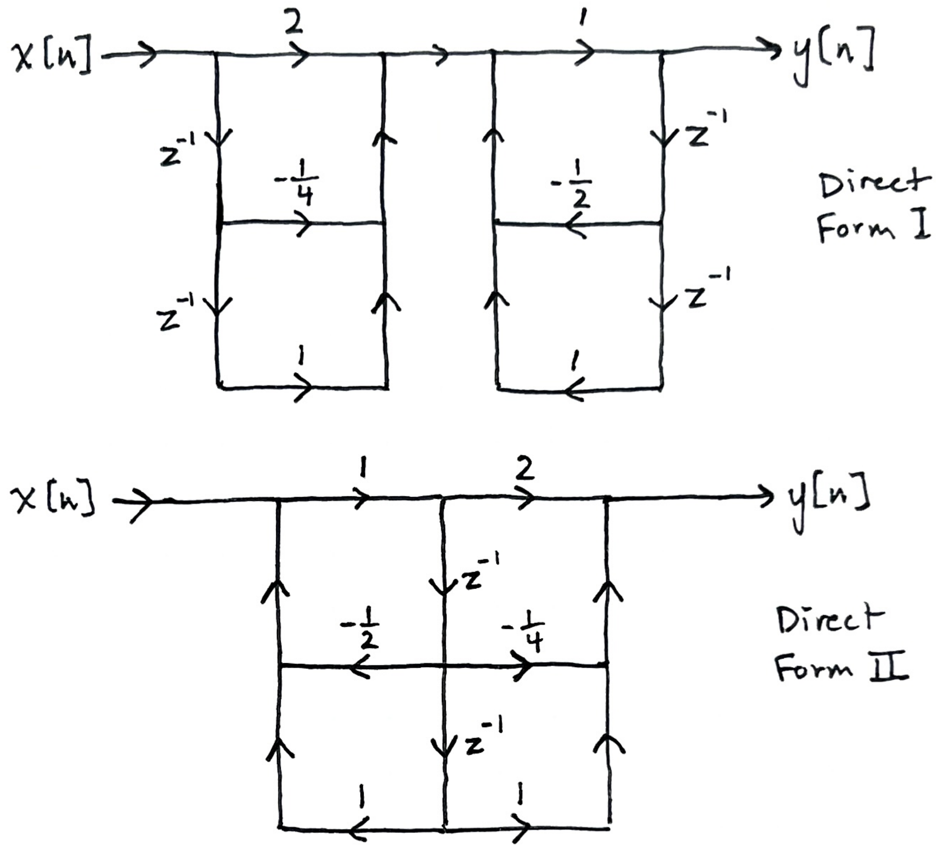

Figure 1: Direct Form diagrams for Problem 2

- $A(\omega) = 2\sin\left(\frac{1}{2}\omega\right)$
- $\alpha = \frac{1}{2}$
- $\beta = \frac{\pi}{2}$

The filter does **not** have linear phase because $\beta \neq 0$. (Also, $A(\omega)$ is not always non-negative).

(b)

Since the filter has no symmetry, it **is not a GLP filter**.

(c)

The filter is $h[n] = \{1, 0, 0, 0, 1\}$. The length is $N = 5$ (odd). We check for symmetry about $n = 2$:

- $h[0] = 1$, $h[N-0-1] = h[4] = 1$. ($h[0] = h[4]$)
- $h[1] = 0$, $h[N-1-1] = h[3] = 0$. ($h[1] = h[3]$)

Since $h[n] = h[N-n-1]$, the filter has **even symmetry**. An odd-length filter with even symmetry is a Type-I filter. Therefore, it **is a GLP filter**.

The frequency response is:

$$\begin{aligned} H(\omega) &= 1 + e^{-j4\omega} \\ &= e^{-j2\omega}\left(e^{j2\omega} + e^{-j2\omega}\right) \\ &= 2\cos(2\omega)e^{-j2\omega} \end{aligned}$$

Comparing this to $H_d(\omega) = A(\omega)e^{j(-\alpha\omega+\beta)}$:

- $A(\omega) = 2\cos(2\omega)$
- $\alpha = 2$
- $\beta = 0$

The filter does **not** have linear phase. Although $\beta = 0$, the real-valued amplitude response $A(\omega)$ is not always non-negative (it changes sign, which introduces π jumps in the phase). A true linear phase response requires $\angle H_d(\omega) = -\alpha\omega$, which means $A(\omega)$ must always be non-negative.

(d)

The filter is $h[n] = \{1, 1, 1, 1\}$. The length is $N = 4$ (even). We check for symmetry about $n = 1.5$:

- $h[0] = 1$, $h[N-0-1] = h[3] = 1$. ($h[0] = h[3]$)
- $h[1] = 1$, $h[N-1-1] = h[2] = 1$. ($h[1] = h[2]$)

Since $h[n] = h[N-n-1]$, the filter has **even symmetry**. An even-length filter with even symmetry is a Type-II filter. Therefore, it **is a GLP filter**.

The frequency response is:

$$\begin{aligned} H(\omega) &= 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} \\ &= e^{-j\frac{3}{2}\omega}\left(e^{j\frac{3}{2}\omega} + e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega} + e^{-j\frac{3}{2}\omega}\right) \\ &= \left(2\cos\left(\frac{1}{2}\omega\right) + 2\cos\left(\frac{3}{2}\omega\right)\right)e^{-j\frac{3}{2}\omega} \end{aligned}$$

Comparing this to $H_d(\omega) = A(\omega)e^{j(-\alpha\omega+\beta)}$:

- $A(\omega) = 2\cos\left(\frac{1}{2}\omega\right) + 2\cos\left(\frac{3}{2}\omega\right)$
- $\alpha = \frac{3}{2}$
- $\beta = 0$

The filter does **not** have linear phase. Although $\beta = 0$, the amplitude response $A(\omega)$ is not always non-negative (it can change sign).

(e)

This filter appears to have odd symmetry. However, for a Type-III (odd length, odd symmetry) filter, the center point **must** be zero. Here, the center index is $n = 1$ and $h[1] = -1 \neq 0$. Therefore, the filter does not satisfy the conditions for Type-III symmetry. Since the filter has no symmetry, it **is not a GLP filter**.

Grading: *20 points, 4 points each*

- +2 point: correct $A(\omega)$
- +1 point: correct α
- +1 point: correct β

  OR

- +4 points: correct identification for not GLP with justification

4

Solution:

(a)

From the plot of $\angle H_d(\omega)$, we can see that the phase has a constant slope, but it jumps by π at a few points. If the filter had strictly linear phase, we would only see jumps by 2π present. Thus, the filter **does not have strictly linear phase**.

(b)

Recall that any GLP filter will have a DTFT of the form:

$$H_d(\omega) = A_d(\omega)e^{j\Psi(\omega)} \tag{2}$$

$$\Rightarrow \angle H_d(\omega) = \Psi(\omega) + \angle A_d(\omega) \tag{3}$$

Above, $A_d(\omega)$ is the real-valued amplitude response and $\Psi(\omega)$ is the complex phase term. In order for this filter to be a Type-III or Type-IV filter, it would need to satisfy

$$\Psi(\omega) = -\alpha\omega + \frac{\pi}{2} \tag{4}$$

for some real constant α. Since $A_d(\omega)$ is real-valued, $\angle A_d(\omega)$ can only ever equal 0 or π (same as $-\pi$). This means that at $\omega = 0$, $\angle H_d(0)$ can only equal $\frac{\pi}{2}$ or $-\frac{\pi}{2}$ for a Type-III or Type-IV filter.

Meanwhile, the plot shows $\angle H_d(0) = 0$. Therefore, the filter **cannot be a Type-III or Type-IV filter**.

(c)

Recall that the slope of the phase of a linear phase filter is equal to the midpoint of the impulse response. Thus, we can determine N by finding the slope of the phase plot and using the equation:

$$\alpha = \frac{N-1}{2} \tag{5}$$

Ignoring jumps in phase, we see from the plot that $\angle H_d(\omega)$ has a slope of -4 everywhere, giving us $\alpha = 4$ and subsequently $N = 9$.

(d)

In the phase plot of a linear phase filter, jumps in phase by $\pm\pi$ correspond to values of ω where $H(\omega)$ changes sign. Therefore, $|H_d(\frac{\pi}{8})| = 0$.

Grading: *30 points (6, 9, 9, 6)*

- (a)
  - $+3$: Correctly identifies not strictly linear phase
  - $+3$: Correct justification
- (b)
  - $+3$: Correctly identifies filter cannot be Type-III or Type-IV
  - $+6$: Correct justification
- (c)
  - $+3$: Uses correct equation to solve for N
  - $+6$: Correct value for N
- (d)
  - $+6$: Correct value for $|H_d(\frac{\pi}{8})|$
    ($+3$ if student recognizes that $H(\omega)$ changes sign at $\omega = \frac{\pi}{8}$ but finds incorrect value)