UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Spring 2026

Homework 9

Profs. Snyder, Shomorony **Due: Friday, Apr. 3, 2025, 11:59pm on Gradescope**

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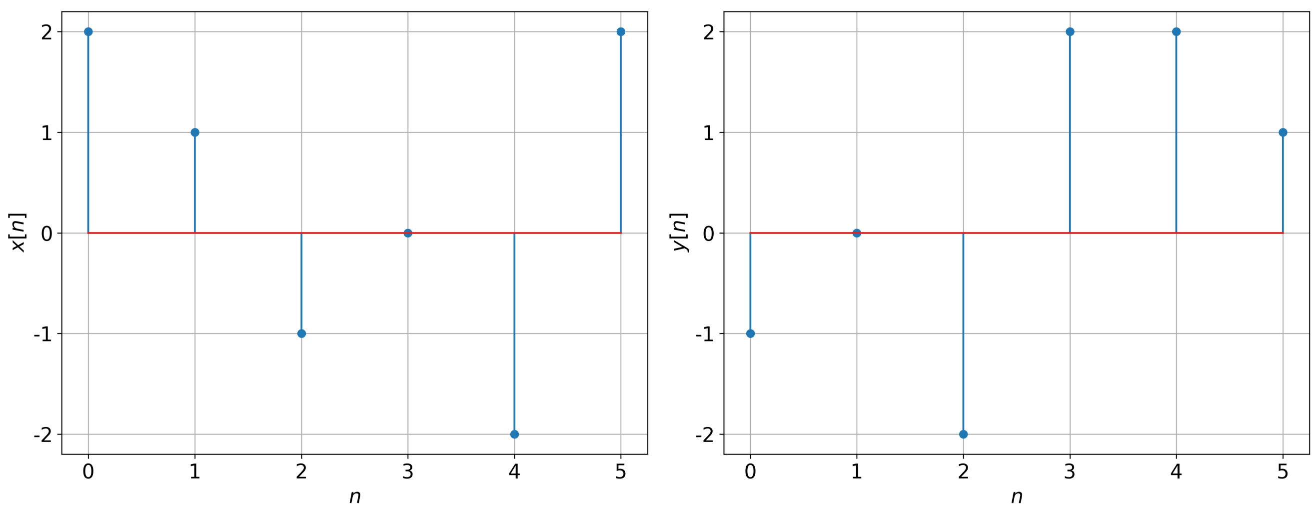
1. The below stem plots show two signals $x[n]$ and $y[n]$. Let $X[k] = \{X_0, X_1, X_2, X_3, X_4, X_5\}$ be the DFT of $x[n]$. Express $Y[k]$ in terms of $X_0, X_1, X_2, X_3, X_4,$ and X_5.



2. Consider a length–8 sequence $\{x[n]\}_{n=0}^{7} = \{x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ with DFT $X[k]$ shown below

$$X[k] = \{A, B, C, D, 0, 0, 0, 0\}$$

where A, B, C, and D are some complex numbers. Suppose we have two new length–8 sequences given by $y[n]$ and $z[n]$. The DFTs of these sequences, $Y[k]$ and $Z[k]$, are shown below.

$$Y[k] = \{0, 0, 0, 0, A, B, C, D\}$$
$$Z[k] = \{A, B, C, D, A, B, C, D\}$$

   (a) Determine the signal $y[n]$ in terms of $x_0, x_1, \ldots, x_7$.

   (b) Determine the signal $z[n]$ in terms of $x_0, x_1, \ldots, x_7$.

3. Let $x[n]$ be a length–18 discrete-time signal with DFT $X[k]$ and DTFT $X_d(\omega)$. Suppose we zero-pad $x[n]$ with 12 zeros to obtain $y[n]$ with DFT $Y[k]$ and DTFT $Y_d(\omega)$. Select all of the following relationships that are true and please show your work or justify your reasoning.

   (a) $X[0] = Y[0]$

   (b) $X[1] = Y[1]$

   (c) $X[3] = Y[5]$

   (d) $X_d\left(\frac{\pi}{5}\right) = Y_d\left(\frac{\pi}{5}\right)$

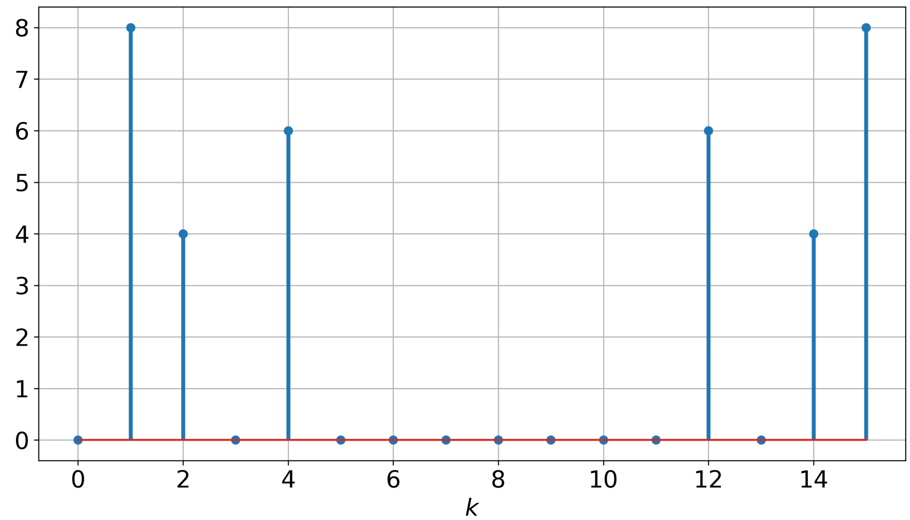
   (e) $X_d\left(\frac{\pi}{5}\right) = Y_d\left(\frac{\pi}{3}\right)$

   (f) $Y[k] = Y_d\left(\frac{\pi}{15}k\right)$

4. A length-16 signal $x[n]$ is known to be given by

$$x[n] = A_1 \cos(\omega_1 n) + A_2 \cos(\omega_2 n) + A_3 \cos(\omega_3 n), 0 \leq n \leq 15,$$

where $\{A_i\}_{i=1}^3$ and $\{\omega_i\}_{i=1}^3$ are the unknown spectral component amplitudes and frequencies, respectively. The magnitude of the DFT $|X[k]|$ is plotted below.



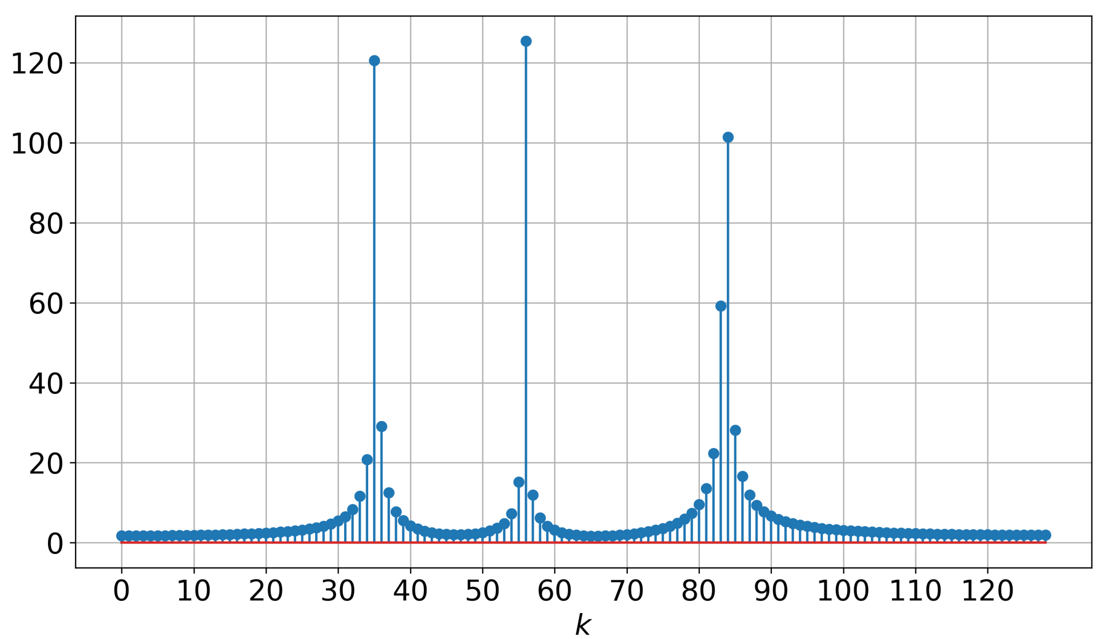
(a) Determine the frequency values ω_1, ω_2, and ω_3. (These frequencies should be between 0 and π).

(b) Determine the amplitude values A_1, A_2, and A_3.

5. Your friend sends you an audio clip they recorded of them simultaneously playing three unknown notes on a piano. This signal $x_a(t)$ may then be expressed as

$$x_a(t) = \cos(\Omega_1 t) + \cos(\Omega_2 t) + \cos(\Omega_3 t)$$

as we assume the three notes have equal amplitudes. You load this audio into Python as a length-256 digital signal $\{x[n]\}_{n=0}^{255}$ with DFT $\{X[k]\}_{k=0}^{255}$. You then generate the magnitude of the DFT $|X[k]|$ in the following plot for $0 \leq k \leq 128$, i.e. values of k corresponding to $0 \leq \omega \leq \pi$.



(a) Identify the three values of k that most closely correspond to the three notes in this audio clip.

(b) Determine the digital frequencies ω_1, ω_2, ω_3 corresponding to your values of k from part (a).

(c) Your friend wants you to figure out which three notes they played on the piano and will only tell you that the sampling rate of the audio clip is $f_s = 1.2$ kHz. Determine the radial frequencies Ω_1, Ω_2 and Ω_3. (You may give exact values or round to three decimal places)

(d) The below table gives two octaves of musical notes and their corresponding linear frequencies (in Hz). Determine the three musical notes your friend played. (The corresponding note should be within 2 Hz of your calculated linear frequencies.)

Note (Octave 3)	C3	C#3	D3	D#3	E3	F3	F#3	G3	G#3	A3	A#3	B3
Frequency (Hz)	131	139	146	156	165	175	185	196	208	220	233	247
Note (Octave 4)	C4	C#4	D4	D#4	E4	F4	F#4	G4	G#4	A4	A#4	B4
Frequency (Hz)	262	277	294	311	330	349	370	392	415	440	466	494