HW8 Solutions

ECE 310: Digital Signal Processing, Spring 2026 Version: **1.0**

Due Date: 3/27/26
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1

Solution:

(a) When we plot $X_a(\Omega)$, we may notice that it can be expressed as the sum of two rectangle functions:

$$X_a(\Omega) = 5\,\text{rect}\left(\frac{\Omega}{100\pi}\right) + 5\,\text{rect}\left(\frac{\Omega}{300\pi}\right)$$

This allows us to use the CTFT pair,

$$\frac{\sin\left(\frac{\tau}{2}t\right)}{\pi t} \longleftrightarrow \text{rect}\left(\frac{\Omega}{\tau}\right),$$

to get

$$\boxed{x_a(t) = \frac{5\sin(50\pi t)}{\pi t} + \frac{5\sin(150\pi t)}{\pi t}.}$$

To determine $x_a(0)$, we use L'Hopital's rule:

$$\begin{aligned}\lim_{t\to 0}\frac{\sin(a\pi t)}{\pi t} &= \lim_{t\to 0}\frac{a\pi\cos(a\pi t)}{\pi} \\ &= a\end{aligned}$$

$$\begin{aligned}\Rightarrow x_a(0) &= (5\cdot 50) + (5\cdot 150) \\ &= \boxed{1000}\end{aligned}$$

Alternatively, we could have used the equivalent transform pair

$$\frac{\tau}{2\pi}\,\text{sinc}\left(\frac{\tau}{2}t\right) \longleftrightarrow \text{rect}\left(\frac{\Omega}{\tau}\right),$$

leading to the equivalent expression

$$\boxed{x_a(t) = 250\,\text{sinc}\,(50\pi t) + 750\,\text{sinc}\,(150\pi t)\,.}$$

Since $\text{sinc}\,(0) = 1$, we can see that $x_a(0) = 1000$, as we found previously.

(b) Given $T = \frac{1}{50}$, we find can find $x[n]$ by setting $t = nT$.

$$\begin{aligned}x[n] &= \frac{5\sin(50\pi nT) + 5\sin(150\pi nT)}{\pi nT} \\ &= \frac{250\sin(\pi n) + 250\sin(3\pi n)}{\pi n}\end{aligned}$$

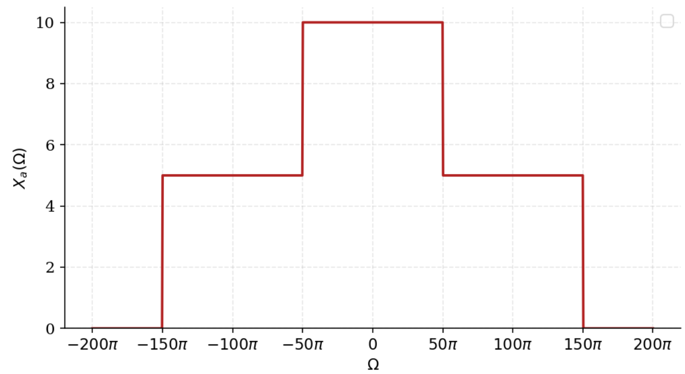

Figure 1: Plot of $X_a(\Omega)$

Notice that $x[n] = 0$ for all $n \neq 0$. However, plugging in $n = 0$ results in an indeterminate form. We know from the previous part that $x_a(0) = 1000$, which means that $x[0] = 1000$. Thus, we can express $x[n]$ as

$$\boxed{x[n] = 1000\,\delta[n].}$$

(c) Applying the DTFT pair

$$\delta[n] \longleftrightarrow 1$$

gives us

$$\boxed{X(\omega) = 1000},$$

so $X(\omega)$ is constant for all ω (See Figure **??**).

Grading: *27 points, 9 points each*

- For part (a):
  - +9 correct
  - +5 correct $x_a(t)$, incorrect or missing $x_a(0)$
  - +4 correct $x_a(0)$, incorrect or missing $x_a(t)$
  - +0 empty or invalid
- For part (b):
  - +9 correct ($x[n]$ expressed as delta or piecewise function)
  - +6 correct, except coefficient is incorrect
  - +0 empty or invalid
- For part (c):
  - +9 correct plot with values clearly labeled
  - +5 correct $X(\omega)$, plot is missing
  - +0 empty or invalid

2

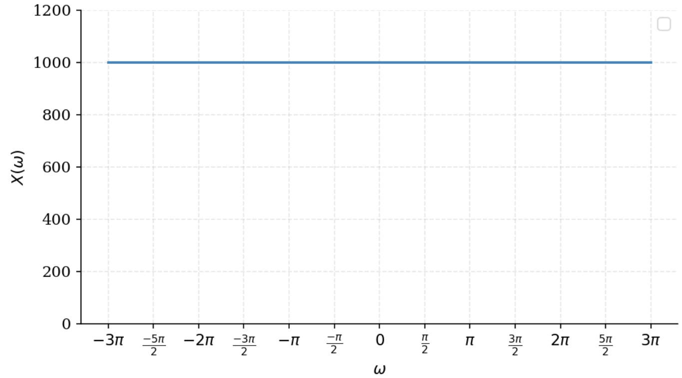

Figure 2: Plot of $X(\omega)$

Solution:

(a) $x_a(t) = \cos(150\pi t)$

Setting $t = nT$ gives us

$$\begin{aligned} x[n] &= \cos(150\pi nT) \\ &= \cos\left(\frac{150\pi}{100}n\right) \\ &= \cos\left(\frac{3\pi}{2}n\right) \qquad (*) \end{aligned}$$

Since ideal D/A conversion removes all frequencies outside of $\omega \in [-\pi, \pi]$, we should express the frequency of the cosine to be within that range, which we can do because $\cos\left(\frac{3\pi}{2}n\right) = \cos\left(-\frac{\pi}{2}n\right)$ due to the 2π-periodicity of the DTFT:

$$\begin{aligned} x[n] &= \boxed{\cos\left(-\frac{\pi}{2}n\right)} \\ &= \cos\left(\frac{\pi}{2}n\right) \qquad (*) \end{aligned}$$

Applying $\Omega = \frac{\omega}{T}$, we get

$$\begin{aligned} y_a(t) &= \boxed{\cos(-50\pi t)} \\ &= \cos(50\pi t). \qquad (*) \end{aligned}$$

(b) $x_a(t) = \cos(100\pi t)$

Setting $t = nT$:

$$\begin{aligned} x[n] &= \cos(100\pi nT) \\ &= \boxed{\cos(\pi n)} \\ &= (-1)^n \qquad (*) \end{aligned}$$

$$\Rightarrow \boxed{y_a(t) = \cos(100\pi t)}$$

(c) $x_a(t) = \sin(100\pi t)$

Setting $t = nT$:

$$\begin{aligned} x[n] &= \sin(100\pi nT) \\ &= \sin(\pi n) \qquad (*) \\ &= \boxed{0} \end{aligned}$$

$$\Rightarrow \boxed{y_a(t) = 0}$$

Notice how just like in part (b), we sampled exactly at the Nyquist rate, yet here we are not able to recover the original signal. This has to do with the difference between the DTFTs of $\cos(\omega_0 n)$ and $\sin(\omega_0 n)$.

All $(*)$ are acceptable answers.

Grading: *21 points, 7 points each*

- For each part:
  - +7 correct
  - +4 correct $x[n]$, incorrect or missing $y_a(t)$
  - +3 correct $y_a(t)$, incorrect or missing $x[n]$
  - +0 empty or invalid

3

Solution:

(a) $x_a(t) = \sin(880\pi t)$, $T_1 = \frac{1}{8000}$

Setting $t = nT$:

$$\begin{aligned} x[n] &= \sin(880\pi n T_1) \\ &= \boxed{\sin\left(\frac{11\pi}{100} n\right)} \end{aligned}$$

To find the length of $x[n]$, N, we divide the duration of the recording, L_{in}, by the sampling period:

$$N = \frac{L_{\text{in}}}{T_1} = \frac{4}{\frac{1}{8000}} = \boxed{32000 \text{ samples}}$$

(b) $y_a(t) = \sin(1760\pi t)$

To solve for T_2, we set up the equation $T_2 = \frac{\omega_0}{\Omega_0}$, where ω_0 is the starting digital frequency and Ω_0 is the target analog frequency:

$$\begin{aligned} T_2 &= \frac{11\pi}{100} \cdot \frac{1}{1760\pi} \\ &= \boxed{\frac{1}{16000}} \end{aligned}$$

We rearrange the same equation from before to determine the duration of the output:

$$L_{out} = NT_2 = \frac{32000}{16000} = \boxed{2 \text{ seconds}}$$

(c) $y_a(t) = \sin(392\pi t)$

Following the same process as part (b):

$$T_2 = \frac{11\pi}{100} \cdot \frac{1}{392\pi}$$
$$= \boxed{\frac{11}{39200}}$$

$$\Rightarrow L_{out} = NT_2 = \frac{11}{39200}(32000) = \frac{440}{49} \approx \boxed{8.98 \text{ seconds}}$$

Grading: *21 points, 7 points each*

- For each part:
  - +7 correct
  - +4 only one answer correct
  - +0 empty or invalid

4

Solution:

(a) From the plot, we can see that $\Omega_{max} = 2\pi B = 80\pi$. Thus, the Nyquist frequency is $f_{\text{nyq}} = 2B = \boxed{80 \text{ Hz}}$.

(b) $T = \frac{1}{180}$

Since the signal was sampled above the Nyquist frequency, there is **no aliasing in** $X_d(\omega)$. In addition, aliasing effects can only be introduced through A/D sampling, so naturally there is **no aliasing present in** $Y_d(\omega)$.

(c) $T = \frac{1}{60}$

Since the signal was sampled below the Nyquist frequency, **there is aliasing in** $X_d(\omega)$. However, the regions where the spectral copies overlap (aliasing) are filtered out by $H_d(\omega)$, so there are **no aliasing effects present in** $Y_d(\omega)$.

(d) Consider the fact that the width of each spectral copy in $X_d(\omega)$ depends on T. Our goal is to get the regions of overlap produced by the sampling process to be exactly the regions that get filtered out by $H_d(\omega)$. Thus, one way to approach the problem is to make the left edge of the right spectral copy lie directly on the right edge of $H_d(\omega)$. We can set this up as an equation and solve:

$$2\pi - \omega_{max} = \frac{\pi}{3}$$
$$2\pi - \Omega_{max}T = \frac{\pi}{3}$$
$$2\pi - 80\pi T = \frac{\pi}{3}$$

Solving for T gives us

$$\boxed{T = \frac{1}{48}.}$$

Grading: *31 points, part (a): 4 points, parts (b) through (d): 9 points each*

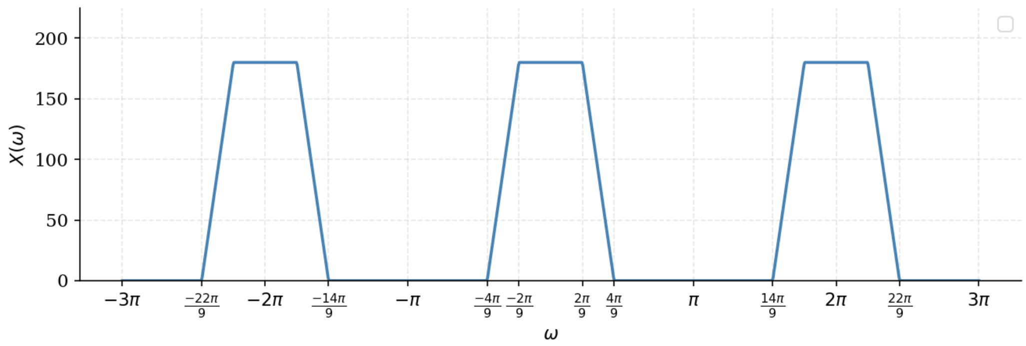

Figure 3: $X(\omega)$, part (b)

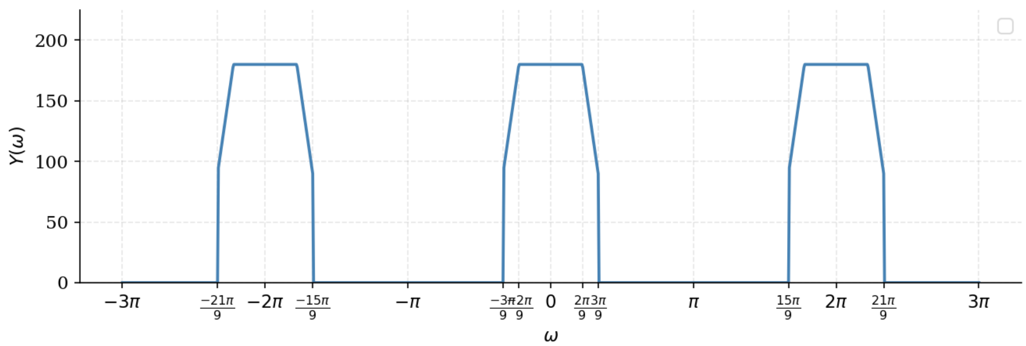

Figure 4: $Y(\omega)$, part (b)

- For part (a):
  - +4 correct
  - +0 empty or invalid
- For parts (b) and (c):
  - +9 correct
  - +6 only one plot incorrect
  - +6 correct plots, incorrect evaluation of aliasing presence
  - +3 correct evaluation of aliasing presence, both plots incorrect
  - +0 empty or invalid
- For part (d):
  - +9 correct
  - +6 correct reasoning, incorrect value of T
  - +0 empty or invalid

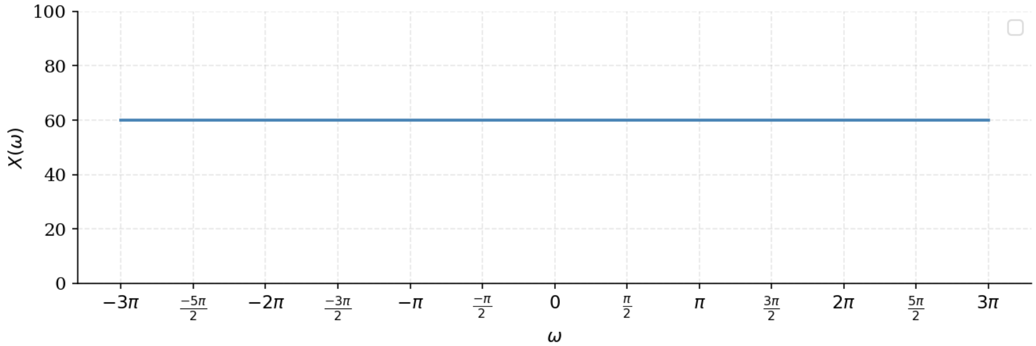

Figure 5: $X(\omega)$, part (c)

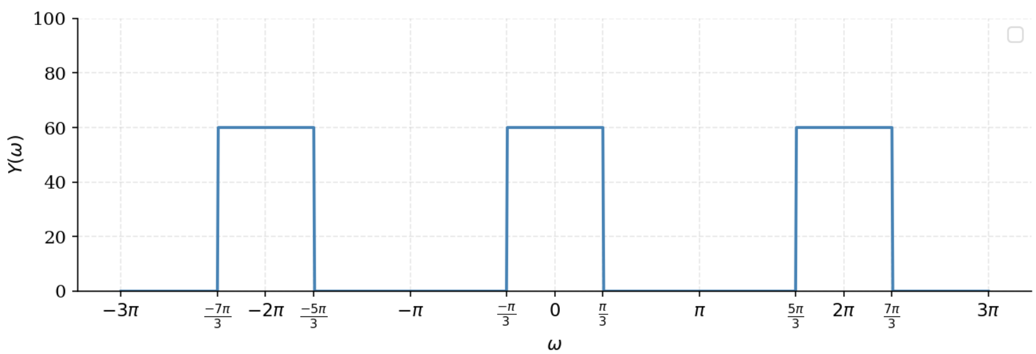

Figure 6: $Y(\omega)$, part (c)